A Manual of Chemical Nomography

—BY—

HORACE G.^{rove} DEMING, PH. D.
Associate in Chemistry
UNIVERSITY OF ILLINOIS

THE UNIVERSITY PRESS
CHAMPAIGN, ILLINOIS
1918

PREFACE

The popularity of any new calculating device is likely to depend very much on the readiness with which the directions can be understood by readers of limited mathematical training. An effort has therefore been made to have the directions accompanying the nomon as simple and explicit as possible. Most people cannot read numbers from a decimally-divided scale without being taught to do so; and the process of locating decimal points by inspection is a mystery to most men who lack a technical education. The reader who finds the tone of this manual too elementary is requested to remember that technical men frequently turn over their routine calculations to less highly trained assistants; and that many persons who grow to full physical stature, remain to the end of their lives in their mathematical infancy. It is believed that a technical expert will be able to run through the directions that accompany the nomon in about an hour, and will use the chart skilfully from the beginning. The novice will need to practice carefully and will acquire skill slowly. Let him have patience. Most things worth knowing are not to be comprehended at a glance.

It is hoped that graphical methods of calculation, thus simplified, may become familiar to many persons who have hitherto been content to multiply and divide by methods that have come down to us from the Middle Ages. It would be hard to name a profession so non-mathematical in its nature that its practitioners have no interest in a device for lessening the labor of simple arithmetical calculations. Even poetry is perhaps to be considered as a branch of applied mathematics; for are not poets said to speak in numbers? Music and painting, too, have their numerical aspects, and their devotees ought perhaps to prefer graphical methods of calculation; for are not musicians and painters all familiar with *scales* of tone and color?

The plan is therefore to issue separate editions of the nomon, in which its applications to the different professions are discussed by specialists. Editions for accountants and for chemists are at this moment in press, and others are in preparation. The author wishes to express his appreciation of the cordial good-will with which the editors of these editions have undertaken the work of adaptation. He invites similar coöperation from those who think that they can still further extend the nomon's field of usefulness.

All the computations employed in the construction of the chart were carried out with a mechanical calculating machine, and checked and cross-checked by repeated differencing to differences of the third order. The standard nomon is issued in 18 sections, consisting of 234 scales, with about 40,000 separate points of subdivision. All these were laid off accurately on metal with a dividing engine, and it is safe to say that errors in the construction of any single scale, amounting to more than one five-hundredth of an inch, have been definitely excluded.

Not quite so high a standard of precision was possible in adjusting the position of each scale with respect to its neighbors. The inaccuracy here in a few cases may amount to several times that stated above, becoming barely visible in the most precise work, and introducing an error of perhaps one part in five thousand at most. Even this source of occasional trifling error will doubtless be overcome by correction of the plates before many copies are issued.

Experience will doubtless result in improvements in the quality of the paper employed and in the transparent preparation used to protect the surface of the charts. Certain materials known to be admirably suited for this purpose at the present moment are quite impossible to obtain. Criticisms and suggestions from the reader will be welcomed by the author, for no useful device ever yet sprang into ex-

istence fully developed, as did Minerva. Rather is mechanical invention like a pearl, growing by slow accretions of thought about a central idea.

Acknowledgment is gratefully made at this time to several friends whose criticisms have helped to make the directions more easily understood; and especially to the author's father, Mr. Joseph J. Deming, whose assistance and encouragement in the early stages of this enterprise had much to do with its final success. It was he who drew the first model of the nomon, entirely by hand, laying off the scales of four sections, point by point, with the greatest accuracy and patience.

H. G. D.

Urbana, Illinois
Jan. 1, 1918.

CONTENTS

APPENDIX

A Manual of Chemical Nomography

Introduction

A group of graphical methods with which chemists have long been familiar is that in which the compositions of mixtures with respect to two or more chemical components are represented by points having definite positions in a plane or in space. Such charts have frequently been referred to as "phase-rule diagrams." A better name for them would be "composition diagrams," since the facts they are intended to bring out concerning substances of varying composition do not necessarily involve questions of phase relationships.

In contrast with such graphical methods for the presentation of experimental facts, which constitute what may be called "Graphical Chemistry," we have a group of methods concerned rather with the solution of numerical problems arising in chemical work. This is the subject-matter of what we propose to call "Chemical Nomography."

Strictly defined, nomography would include only such methods of graphical calculation as employ a single chart, constructed in the beginning, for a whole series of related problems; and would exclude the processes of graphical arithmetic, in its narrower sense, in which each problem is solved by a separate chart, drawn step by step as the computation proceeds, arithmetical operations being translated into their graphical equivalents. The methods of Graphic Statics all belong in this class. But practically there is little advantage in such a distinction, and the reader will discover that certain useful methods are partly nomographical and partly arithmographical.

For the modern development of the subject of nomography credit must be given first of all to d'Ocagne; who in his "*Traité de Nomographie*," and in several more recent works, has given a complete outline of the subject. In German we have the writings of Mehmke, the translation of whose monograph on numerical calculations, in the "*Encyclopédie des Sciences Mathématiques*," with additions by d'Ocagne himself, is perhaps the best exposition of the subject in a small compass. Several works of less importance exist in German, French, Italian and Spanish. In English nothing of the scope of the French treatises has yet been published, though there is an elementary book by Peddle, for the use of engineers, and a series of lectures by Runge, beside a little material in collections of mathematical and engineering papers (for the alinement method see Strachan, *Trans. Am. Soc. Civ. Eng.* 58, 1915).

One is able to discover in chemical literature a number of graphical solutions of numerical chemical problems, but mostly of an undesirable sort. Chemical writers have generally appeared to be unaware of the remarkable recent progress in Nomography, and have based their solutions on the meager inspiration to be drawn from ordinary Analytical Geometry. As a matter of fact one is not likely to discover new or useful methods by any such means. Nomographical science passed that stage of development a generation ago.

The numerical operations called for in routine chemical calculations are almost exclusively multiplication or division, and the summation of series. Though

all the existing nomographical treatises have been carefully reviewed during the preparation of this manual, in a search for ideas likely to be of use in chemistry, the methods finally worked out for the solution of these two most important types of problems are so different from anything previously published as to be considered new in principle. They are nevertheless related to the well-known alinement method, as will be indicated hereafter.

The nomon, or nomographic reckoner, is not intended as a cheap substitute for any other calculating device, but is an independent instrument, having its own field of usefulness. Its superiority over the slide-rule, for example, is not so much to be found in its inexpensiveness or increased accuracy, though these are noteworthy advantages, but in the readiness with which it may be adapted to all manner of special requirements. The reader should bear this fact in mind from the beginning, and should be alert for opportunities to use the nomon in problems other than those involving simple multiplication and division.

The fourth figure of a result read from a nomon is usually somewhat in doubt. A doubtful figure is, however, better than no figure at all, because errors with an instrument of this precision do not accumulate sufficiently to affect the third place of the result of a series of several consecutive multiplications and divisions, as would be the case with a slide-rule, for example. In case unusual accuracy is wanted, the calculator should make use of the precision methods of multiplication and division, described hereafter.

Among the most important of the chemical problems that may be solved with the aid of the nomon are those dealing with mixtures. Frequently such problems may be solved directly; but where the adaptation is difficult the reader is advised to construct for himself a mixture-diagram, based upon the so-called principle of alligation.

Problems in the summation of series may be solved graphically by a number of different methods. The solution hereafter described, based on the use of reciprocal cross-section paper, has not been previously published. The commercial chemist will find that this device supplements the nomon very nicely, the two together being sufficient to cover nearly all the problems arising in his work.

The arrangement of the material presented in the following pages is practical, rather than logical. Since most of the problems met by the chemist may be solved with a nomon, a description is first presented of the use of this device in the simple operations of multiplication and division. Afterward more complex applications are taken up, and finally problems whose solution calls for the use of entirely different methods.

Preliminary Tests

With each chart is furnished an index, consisting of a fine, straight line, ruled on the lower surface of a strip of celluloid or other transparent material. Unfortunately many of these strips shrink a trifle, and the hair-line may be found to deviate slightly from a condition of absolute straightness by the time it reaches the purchaser, even though perfectly straight when it left the factory. The reader is therefore advised to prepare his own index by ruling a very fine, straight scratch on a blank strip with a sharp needle-point, staining the scratch with a drop of ink. This line will probably remain straight for several months. If it ever changes, a new one may be ruled beside it and stained a different color.

The index must next be tested. Lay it across the second section of the nomon, to pass exactly through the point numbered 100 in the upper right-hand corner, and the point numbered 120 in the lower left-hand corner. All the scales in the middle of the page must now be intersected precisely at 24, 36, 48, etc. Now turn the strip around, end for end, and repeat. If the two sets of readings agree, the line is straight; if not, a new one must be ruled. Repeat this test every few weeks.

Some persons prefer to use a black silk thread for lining up values on the chart. This method works well with charts printed on cardboard, and one can acquire considerable dexterity with a little practice; but the process is after all a trifle clumsy, unless one has a case of multiplication by a constant multiplier. It takes more of a tension than one might think to keep the thread straight, and error is apt to result.

If the surface of the paper on which the charts are printed does not lie absolutely flat, no harm is done, provided the paper is not stretched, for the reason that the flexible transparent strip will assume the curvature of the surface against which it is pressed, and three points otherwise in a straight line will remain in curvilinear alinement. Unlacquered sections may therefore be rolled up loosely, *front surface on the outside,* for convenience in carrying, without any damage to the charts.

Shrinkage of the paper on which the charts are printed will not affect the accuracy of results, even though shrinkage in the vertical direction is not equal to that in the horizontal direction, provided both horizontal and vertical shrinkage be uniform in all parts of the chart. This is almost always the case; in fact the only instance of irregular shrinkage that has come to the notice of the writer, was one in which the chart was placed on a radiator, with deliberate intent to distort it. One way of testing the chart for errors due to shrinkage is to connect the top and bottom of the inclined scale at the right of the chart, and every numbered interval between these limits, *by means of an index known to be straight,* with the top or bottom of the scale at the left-hand margin. The nine scales near the middle of the page must now be intersected precisely at points corresponding to whole numbers.

How to Read the Chart

Before making use of the chart in actual calculations it will be well for the reader to make sure that he can read numbers from the different scales, at high speed, without making mistakes. This is easy with scales in which each whole unit is numbered, tenths of a unit being indicated directly, as with the scales numbered 1 to 8 inclusive on the second section. In this case it is even possible to estimate hundredths of a unit.

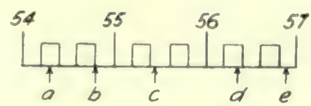
Thus connect the point 127.4 on the scale at the left-hand margin of the chart on the second section with the point 50 on the inclined scale at the right, by means of a transparent index, previously tested for straightness, as described on the preceding page. The scale numbered 1, just to the left of the inclined scale, is inter-

sected between 19.1 and 19.2; and, as nearly as one can estimate, at a point one-tenth of the distance from the former to the latter. The reading here is therefor 19.11. Without moving the transparent strip, read the intersections with the six scales just to the left of scale 1. These readings are respectively 31.85, 44.59, 57.33, 70.07, 82.81 and 95.55.

The case is similar when every *tenth* unit of the scale is numbered, whole units being indicated directly. Thus turn to the eighteenth section of the chart, and connect the point 941 on the scale at the left with the point 50 on the inclined scale. The scales numbered 6 and 7 are intersected in the points 611.7 and 705.8 respectively. The decimal figure in these two examples may be a trifle in doubt, since the distance between the points representing consecutive units is very small.

Notice that a unit of the inclined scale of each section of the chart is divided into halves, tenths of a unit being estimated visually. Thus connect the point 155 on the scale at the left of the second section with the points 20, 21, 22, and 23 on scale 1. The inclined scale is intersected at 29.0, 35.5, 41.9 and 48.4, respectively.

Each unit of the scale numbered 9 on the second section is divided into fifths. Tenths of a unit may still be estimated. The odd numberd tenths fall in the middle of the blocks or teeth that intervene between consecutive units, and the even numbered tenths at the edges of these blocks. Thus the point *a* in the sketch below corresponds to 54.3, and the point *b* to 54.8. The point *c* corresponds to 55.45, since it lies half way between 55.4 and 55.5. In favorable cases it may even be possible to split up each block by visual estimation into five equal parts, each corresponding to two one-hundredths. Thus the point *d* in the annexed sketch corresponds to 56.34, it being seven-tenths of a block, or fourteen-hundredths of a unit, beyond 56.2; similarly the point *e* corresponds to 56.88. It is hardly possible to carry this process of visual interpolation any further, and readings from scales in which the units are divided into fifths will in general be in doubt by one or two hundredths of a unit. Most of the errors committed in reading numbers from the chart occur in connection with scales of this description, and the reader of limited graphical experience will do well to develop skill in reading such scales before passing on. Practice will be furnished by throwing the transparent strip across the eighth section of the chart, in random positions, reading the intersections with the different scales.



Multiplication

To multiply two numbers together, find one of them, paying no attention to the decimal point, on the left-hand scale of one of the sections of the chart. This will hereafter be referred to as the *principal scale*. Find the other number, *after dropping its first figure*, on the *inclined scale* at the right of the chart. Lay the transparent strip, tested for straightness as previously described, to pass through the two points thus located. The product will be read where the scale is intersected that is labeled above and below with the figure of the multiplier that has been dropped. These nine numbered scales will hereafter be referred to as *intermediate scales*. Thus the product of 146.8 and 441.9 is found by locating 146.8 on the scale at the *left* of the third section, and 41.9 on the inclined scale at the right. If the index is placed to connect these two points, it will intersect the scale numbered 4 (the figure dropped) in the point 64.87. The decimal point is located by noting that 146.8×441.9 is about the same as $150 \times 400 = 60{,}000$. The result given by the chart is therefore pointed off to read 64,870.

It is always possible to locate the decimal point of the result of a multiplication or division by a rough calculation, as above, but there is a simple rule that may

be used instead. The total number of figures preceding the decimal points of multiplier and multiplicand is the same as the number of figures preceding the decimal point of the product, *with the exception noted in the following paragraph.* Thus multiply 146.8 by 741.9. The chart gives 108.9. Since there are three figures preceding the decimal point in both multiplier and multiplicand, there will be $3+3=6$ figures preceding the decimal point of the product, which is therefore pointed off to read 108,900.

The exception to this rule referred to above occurs *when but two figures are found in the number that is printed on the chart just below the place where the product is read.* Thus multiply 146.8 by 415.7. The chart gives the product 61.02. In this case the number printed on the chart just below the place where the product is read is 61, which has but two figures. By reference to other sections of the chart the reader will note that this is one less than the usual number. The product has therefore but five figures, instead of $3+3=6$ indicated by the rule, and is accordingly pointed off to read 61,020.

Since the decimal point of a product can always be located by this rule, it is best to neglect the decimal points of quantities to be multiplied together, treating all of them as if they were numbers lying between 100 and 1000. Thus 1.71, 17,100, and 0.000171 all correspond to the point on the principal scale marked 171; or, if any one of these numbers is to be multiplied by a quantity already located on the principal scale, it will correspond to the point marked 71, on the inclined scale at the right of that particular section of the chart, the product being read from the intermediate scale numbered 1.

It is therefore best to read a number to be looked up on either the principal scale or the inclined scale as if it were composed of a single figure, followed by two groups of two figures each, no matter what the position of the decimal point. Thus 81.39 is read "eight—thirteen—ninety", and 0.010717 is read "one—nought seven—seventeen". The first figure indicates the section of the chart on which the number is to be located or the intermediate scale from which the product is to be read. The following group of two figures gives the whole division, and the last group the fractional part of a division on the scale. If one falls into the habit of calling a number off to himself when about to turn to the chart to search for that number, there is a much smaller chance for making mistakes.

Let us next undertake the multiplication of 14.82 by 10.47. These must both be considered as numbers lying between 100 and 1000, *viz.* 148.2 and 104.7 respectively. Find the first of these two numbers of the principal scale, and 04.7 (not 47.) on the inclined scale between 00 and 10; connect these two points by the index. The figure of the multiplier that has been dropped is 1, hence the result, 15.52 is to be read from the intermediate scale numbered 1.

Now since each of the two numbers to be multiplied together has two figures preceding the decimal point, the number of figures that precede the decimal point of the product would ordinarily be $2+2=4$. But in this case the number printed on the chart just below the place where the product is read is 17, which has only two figures. The decimal point of the final result is therefore to be moved one place to the left from the position it would otherwise occupy, hence the final result is 155.2 The decimal point in this case might have been located more conveniently by inspection, since $15\times10=150$.

The position of the decimal point is determined in a similar way when both the numbers to be multiplied together are decimals. The number of zeros following the decimal point in the product will be equal to the total number of zeros immediately following the decimal points of the two numbers to be multiplied together, provided three figures appear in the number printed on the chart just below the

place where the product is read—otherwise the decimal point of the product will have to be moved one place to the left, as before.

Let us find the product of 0.9368 and 0.0007417. We locate 936.8 on the principal scale (section 18) and 41.7 on the inclined scale. The product, 694.8, is read on scale 7. Now since the number 690 (three figures) is printed on the chart just below the point where this result is read, the number of zeros immediately following the decimal point of the product will be equal to the total number of zeros following the decimal points of the two numbers multiplied together, *viz* $0+3=3$. The final result is therefore pointed off to read 0.0006948.

On the other hand, the product of 0.08482 and 0.01074 is read from the chart (section 17) at a point just over 80, a number with only two figures. The decimal point of the product, which would otherwise have had two zeros following it, must accordingly be moved one place to the left, giving the result 0.000888.

When one of the two numbers to be multiplied together is a decimal fraction, and the other is a whole number, the zeros immediately following the decimal point of the former are cancelled off against the figures preceding the decimal point in the latter to get the number of zeros following or figures preceding the decimal point in the product, as the case may be—provided always that three figures appear in the number printed on the chart just below the place where the product is read.

Thus the product of 93.68 and 0.0007417 is read from the chart (section 18) immediately over the number 690 (three figures). There will accordingly be $3-2=1$ zero immediately following the decimal point of the product, which is accordingly 0.06948. But the product of 8.482 and 0.1047 is read from the chart immediately over the number 80 (two figures). The decimal point is therefore moved one place to the left, from the position it would otherwise occupy, giving the final result 0.888. Similarly the product of 84820 and 0.01047 is 888.0. One should fall into the habit of checking decimal points located in this way by a rough calculation.

When two numbers are to be multiplied together it is nearly always the best plan to locate the one beginning with the larger figure on the principal scale. The reason is that in this case most of the labor of disconnected multiplications and divisions will fall on the last few pages of the chart, and much of the time otherwise lost in turning leaves will be saved. In the long run, if this precaution is observed, about 70 per cent of the work will be done with the last four sections of the chart, and about 96 per cent with the last ten sections. One has thus, in effect, a chart occupying a much smaller number of pages than would seem to be indicated by the fact that it is divided into eighteen separate sections. A marginal index, moreover, gives the lowest number occuring on the principal scale of each section of the chart, and so makes reference easy.

The attention of the reader should also be directed to the readiness with which mistakes can be made when the second figure of a number to be used as a multiplier is zero. For example, if 7.0501 is to be so used, we drop the 7 and look up 05.0 on the inclined scale, between 00 and 10. To drop both the 7 and the following 0 would give 50.1, an entirely different point on the inclined scale, leading to an incorrect result. Notice also that one needs to observe whether the interval between successive numerals on the scale is divided into units or into tenths of a unit. Thus 30.9 (intermediate scale 1, third section) is the *ninth* indicated subdivision beyond 30; but 630.9 (intermediate scale 6, section 18) falls just short of the *first* indicated subdivision beyond 630. Compare these with points representing 30.09 and 639. Skill in avoiding pitfalls of this sort is obtained by practice in solving problems in which such difficulties have been intentionally introduced. The beginner should work the following examples before proceeding further. Answers will be found in the back of the manual.

Problems

Find the product of :—

(1) 888.8 and 7.094
(2) 401.1 and 0.5555
(3) 0.008007 and 0.6013
(4) 2.6438 and 7.8534
(5) 0.7868 and 0.0099
(6) 9.431 and 1.006
(7) 12.34 and 0.00021
(8) 81345 and 111.1

Precision Multiplication

It is now time to inquire what accuracy is in general to be obtained in multiplication, by the use of the methods that have been described. Referring to the nine numbered intermediate scales on any section of the chart, it will be noticed that the length of the interval between successive subdivisions varies from scale to scale. The accuracy of a product read from one of these scales will therefore depend on the number of that scale, on the section of the chart used, and on the angle at which the hair-line of the transparent index cuts the scale.

In careful use of the chart, the error in reading any scale need not in general be greater than one-fifth of the interval between successive subdivisions, if that interval be very short; and is more likely to be about one-tenth of an interval in the case of scales with points of subdivision relatively far apart. If the intermediate scales numbered 1 and 2 be excluded, results will in general be accurate to within one part in four or five thousand, or within one cent in forty or fifty dollars, with proportionately greater errors in larger numbers.

On the other hand, the error will frequently be not less than one part in one or two thousand, in the case of products read from the intermediate scales numbered 1 and 2. This is the same as saying that multiplication by numbers beginning with 1 or 2, using the methods previously described, is not capable of yielding results of such great relative accuracy. Fortunately a modification of our regular method of multiplication will enable us to get extremely accurate results in this case also. An example or two will serve to make the method clear.

Let us consider the multiplication of 10.47 by 848.2. If the number beginning with the *larger* figure (*i. e.* 848.2) is located on the principal scale (section 17) and the other (after dropping its first figure) on the inclined scale, the product will be found on the intermediate scale numbered 1, and can be read with certainty to three figures only. If the number beginning with the *smaller* figure (*i. e.* 10.47) is the one taken from the principal scale (section 1) the accuracy of the result would be somewhat increased, but hardly enough to make it worth our while to reverse the rule given above.

Instead of following either of these methods let us therefore multiply 848.2 by 10 and then add in the product of 848.2 by 0.47. We have :—

10×848.2 (multiplied mentally)	8482.
0.47×848.2 (read from chart, section 17, scale 4)	398.6
Final result, correct to five figures	8880.6

Again, let us multiply 0.8672 by 0.243. We take 0.2×0.8672 and then add in the product of 0.043 and 0.8672, thus :—

0.2×0.8672 (multiplied mentally)	.17344
0.043×0.8672 (read from chart, section 17)	.03729
Final result, correct to five figures	.21073

Of course the same principle may be extended to multiplication by numbers beginning with other figures than 1 or 2, if a very high degree of accuracy is desired. We shall return to this topic later.

Multiplication by a Constant

Very frequently a number of different quantities all need to be multiplied by the same number. A small hole may be punched with a paper-punch near one end of the transparent strip, exactly over the hair-line. This hole is then set over the point on the principal scale corresponding to the constant multiplicand, and held in place with the rubber tip of a pencil, cut to a conical point. If the free end of the strip is now swung over the face of the chart, the constant multiplicand may be multiplied by each of a series of numbers in turn, and products may be read from the chart almost as fast as an assistant can note them down. This method fails, however, to bring out the full accuracy of the chart, since the hole in the transparent strip is apt not to be precisely centered over the hair-line. The given section of the chart may also be purchased separately, and the hair line set permanently over the constant multiplicand by means of a pin or thumb-tack. Charts printed on reinforced bristol are best suited to withstand this treatment without tearing.

Division

Since division is the converse of multiplication, it may be performed by a method that follows directly from that described for the latter operation. As an example, divide 69,327 by 1.837. Place the hair-line of the transparent strip over the point 183.7 on the principal scale (fifth section). Swing the strip until the hair-line passes through 693.3, which is to be sought in the intermediate scale numbered 3, since the first figure of the quotient is seen to be 3. The inclined scale at the right is now intersected at 77.4, and the quotient is accordingly 37.74. The decimal point is located by noting that 70 : 2 = 35, thus identifying the quotient as a number in the neighborhood of 30.

As a further example, divide 0.88806 by 104.7. Place the hair-line to pass through the point 104.7 on the principal scale (section 1). Swing the free end of the celluloid strip until the hair-line passes through 88.81, which is to be sought on the intermediate scale numbered 8, since that is evidently the first figure of the quotient. The inclined scale at the right is now intersected at 48.2, hence the quotient is 848.2, except for the position of the decimal point, which is still to be determined. But a rough division, 0.8 : 100 = 0.008, indicates the presence of two zeros following the decimal point of the quotient, hence the final result, 0.008482. As a check on the work, make sure that the product of the divisor and quotient is the dividend. In this case, 104.7 multiplied by 0.008482 gives 0.8881, applying the rule previously given for locating the decimal point in multiplication.

It will sometimes happen that the dividend sought on the chart occurs on two different intermediate scales. No error can be committed on account of this fact, however, for if it is located on the wrong scale, and the hair-line passed through it, the latter will be inclined so sharply upward or downward that it will fail to intersect the inclined scale at all. In this case one simply passes over to an adjoining intermediate scale, and there locates the dividend again.

Algebraical Rule

For Locating the Decimal Point

The decimal point of the result of a multiplication or division is best located by a rough calculation or mere inspection. The process becomes easy after a little

practice. But the following rule, which will appeal most to those used to thinking in algebraical terms, may be employed if desired:

Let the number of figures that precede the decimal point of any quantity greater than unity be called its index number; and let the index number of a decimal fraction be the number of zeros that immediately follow the decimal point, with a negative sign prefixed. Thus the index number of 12.34 is 2; that of 0.1234 is 0; and that of 0.001234 is —2.

In multiplication, the index number of the product is the algebraical sum of the index numbers of multiplier and multiplicand; provided three figures appear in the number printed on the chart just below the place where the product is read. Otherwise the decimal point of the product is to be moved one place to the *left*.

In division, the index number of the quotient is the index number of the dividend minus the index number of the divisor; provided three figures appear in the number printed on the chart just below the place where the dividend is read. Otherwise the decimal point of the quotient is moved one place to the *right*.

Problems

Find the quotient of

(9)	5001 ÷ 9.003	(13)	12.007 ÷ 3.142
(10)	0.0003 ÷ 77	(14)	0.10009 ÷ 6.44
(11)	0.1234 ÷ 0.5678	(15)	7.881 ÷ 7.964
(12)	4444 ÷ 0.00449	(16)	1.000 ÷ 3.1416

Precision Division

The method previously given for precision multiplication suggests a similar method for division, which should be used if the first figure of the quotient is 1 or 2, and it seems necessary to reduce the error in the result to less than one part in one or two thousand. Determine the first figure of the quotient by inspection. Multiply the divisor by this figure, neglecting the decimal point, and subtract the result from the dividend. (Up to this point the process is exactly like that of ordinary long division.) Divide the remainder by the divisor, using the chart, and prefix the figure previously obtained by inspection to the quotient read from the chart. Thus to divide 1234 by .05478, notice that the first figure of the quotient must be 2.

Dividend	12340
2 × 5478, multiplied mentally	10956
Remainder	1384

But 1384÷5478 is given by the chart as 2527.

The quotient sought is therefore 22527, correct to five significant figures, the decimal point being located by inspection or by one of the rules previously given (it being noted that three figures appear in the number 220, which is printed on the chart just below the point where the dividend would be found if the ordinary method of division had been used.

In applying this method of precision division, one precaution must be taken if errors are to be avoided. Always take pains to discover whether a zero follows the first figure of the quotient.

Thus let it be required to find the quotient of 3709÷3456.

Dividend	3709
1×3456	3456
Remainder	253

In the ordinary process of long division, the next stage of the calculation would be to bring down a zero from the dividend, then divide 2530 by 3456. But this division is impossible until still another zero is brought down from the dividend, hence the required second figure of the quotient is zero. On dividing the remainder by the divisor, with the aid of the chart, we obtain the result 7321. The complete quotient is therefore 1.07321, correct to six figures.

It is evident that if the remainder had not been inspected to discover the fact that the second figure of the quotient is 0, the final result would have been incorrectly given as 1.7321. The best method of detecting the presence of a zero or zeros following the first figure of the quotient is to repeat the work, using the chart by the ordinary method. This operation both affords a check on the accuracy of the work, and one on the position of the decimal point, and should therefore never be omitted.

Reciprocals

Reciprocals may be read from the chart in the same way that ordinary division is performed, *i. e.* by locating the number whose reciprocal is to be found on the principal scale of one of the sections of the chart, then swinging the free end of the transparent strip until the hair-line passes through 100 on one of the nine numbered intermediate scales. The first figure of the reciprocal is then read from the top or bottom of this intermediate scale, and the three following figures at the point where the inclined scale at the right is intersected.

When the given number begins with 5 or a larger figure, the first figure of the reciprocal will be 1. In this case it may be preferred to find the reciprocal by the precision method of division, which always gives a result to five significant figures. Thus to find the reciprocal of 0.5678, note that the first figure of the quotient, when this number is divided into 100, is 1. We therefore subtract 1×5678 from 10000, decimal points being neglected as is usual in the case of precision division, and divide the remainder (4322) by 5678. The result of this division is 7612, no zeros occurring, hence the required reciprocal is 0.17612, the decimal point being located by inspection or by a rule to be given later.

In order to make this process of finding reciprocals easy, the result of subtracting each of the numbers 500 to 1000 from the number 1000 has been written just outside the principal scale of the six final sections of the chart, forming an additional scale of values, which must in this case be read from above downwards. Thus opposite the number 567.8, whose reciprocal was sought above, may be read the remainder 4322, on the reversed scale (section 14).

To find the reciprocal of any number beginning with 5 or a larger figure, we therefore pivot one end of the transparent strip over the corresponding point on the principal scale of one of the last six sections of the chart. Note the number given by the reversed scale at this point, and find this same number on one of the nine intermediate scales. Swing the free end of the transparent strip until the hair-line passes through the point on the intermediate scale thus located. The first figure of the required reciprocal is 1, the second figure may be read at the top or bottom of the intermediate scale, and the three following figures at the point where the inclined scale is intersected. The result obtained in this way should be verified im-

mediately by dividing the number whose reciprocal has just been found into 100, using the regular process of division. The position of the decimal point is thus established, and the presence of a possible zero or zeros following the first figure of the reciprocal is revealed.

Another check on the position of the decimal point is the rule that the number of zeros immediately following the decimal point of the reciprocal of a number is *one less* than the number of figures that precede the decimal point in the number itself; and conversely. *Powers of ten are an exception to this rule.*

Problems

Find the reciprocals of the following numbers:—

(17) 7.777
(18) 0.00015
(19) 0.7854 (to five figures)
(20) 176.12

Division By a Constant

When a number of different quantities are to be divided by the same divisor, the hair-line of the transparent strip may be pivoted over the point corresponding to the divisor on the principal scale of one of the sections of the chart, by means of a rubber-tipped pencil or a pin. The free end of the strip is then swung over the chart until the hair-line passes through each dividend in turn. The first figure of the quotient may then be read at the top or bottom of the scale on which the dividend is found, and the three following figures of the quotient on the inclined scale at the right.

But there is another method of division by a constant divisor that is somewhat more accurate and therefore to be preferred. Since division by any number is the same as multiplication by its reciprocal, we may find the reciprocal of the constant divisor and use this to multiply each of the dividends in turn. The result is read in each case from the vertical scale that is numbered to correspond with the first figure of the given dividend.

Thus to divide a number of quantities by 0.5678, find the reciprocal of 0.5678 by the method previously described. The result, 0.17612 is next found on the principal scale (section 4) and the hair-line pivoted over it. If the first of the quantities to be divided by the constant divisor is 0.1234, move the free end of the transparent strip until the hair-line passes through 23.4 on the inclined scale, when the required result, 0,2173, will be read on scale 1.

One advantage in substituting a process of multiplication for one of division by the expedient just described, is that numbers can be found a trifle more quickly on the inclined scales than on the nine intermediate scales. Another is that it gives one an entirely independent check on results obtained by the usual process of division. If two sets of results agree, the chance of an error remaining undiscovered is negligibly small.

How to Secure Any Desired Accuracy

The rules previously given for precision multiplication and division may be extended to secure any desired degree of accuracy in the results of these operations. As an example of the method employed in multiplication, let us find the product of π, or 3.14159^{+} and 123.456. The grammar-school methods of multiplication will give the result 387.84813504. But this quantity is expressed far more accurately than is necessary for any practical purpose. What is worse, it is expressed far more accurately than the accuracy of the two numbers multiplied together justifies; for it should be remembered that a product cannot possibly be more accurately known than

the least accurate of the two factors that go to make it up. In the example considered, since the multiplier has been given to but six figures, we are warranted in retaining but six figures in the final result.

Our problem is therefore to find the product of the two numbers given, to six figures, by means of the chart. Now it is evident that six figures are just two figures more than can be read directly from the chart. We must therefore multiply the multiplicand by the first two figures of the multiplier, taken separately, and then add in the product of the multiplicand and the remaining four figures of the multiplier, using the chart. The work is shown below.

	3.14159	
	123.456	
	314.159	(100×3.14159)
	62.8318	(20×3.14159)
	10.858	(3.456×3.1415)
Total	387.8488	

In performing the final multiplication by means of the chart, it is best to take the multiplicand, 3.1415, from the principal scale, and the residue from the multiplier (3.456) from the scales at the right. If this is done, one is enabled to read from the chart the approximate value of the final result to four figures, and thus get a check on the work, without turning to a different section. Thus 3.1415 (principal scale, section 9) $\times$ 123.4 is given by the chart directly as 387.5, checking the more accurate result obtained above.

Even in the cases in which a product or quotient needs to be secured to six or seven figures, the chart is able to save more than half the time needed to calculate the result by non-graphical methods. In reality the saving is greater than this, for the old-fashioned computer always needs to repeat his work in order to make sure he is right. With the nomon the result is verified to four figures in an instant, *by an entirely independent method.*

Note that it is necessary to determine the position of the decimal point of each of the results of multiplication, and arrange the products with the decimal points in a vertical column. The sum of the three partial products is rounded off to six figures, giving the final result 387.849, accurate to within one unit in the last place.

In order to illustrate the process of division, let us divide the number 387.849 by 3.14159·. Evidently the quotient must not be given to more than six, or at most seven figures, since the dividend is known to be uncertain in the sixth place. We obtain two of these six figures (viz. 12) by the ordinary process of long division, neglecting decimal points. We then have a remainder 108582, which is divided by the divisor, using the chart, which gives 3456, or four figures more. The final result is therefore 123.456, the decimal point being located by a rough division, using the chart in the ordinary way to secure the approximate result 123.45. This final check must never be omitted, even if the position of the decimal point is evident from simple inspection, since one or more zeros following the second figure of the quotient might otherwise be overlooked.

This seems to be the proper place to remark that subtraction is best performed by determining mentally what figure needs to be added to each figure of the subtra-

hend in order to give the corresponding figure of the minuend. Thus to subtract 314159 from 387849, the operations performed mentally are:

387849
314159
73690

$9+0=9$ (set down 0).
$5+9=14$ (set down 9, carry 1).
$1+1$ (carried)$=2;+6=8$ (set down 6).
$4+3=7$ (set down 3).
$1+7=8$ (set down 7).

This method will prove useful whenever subtraction needs to be performed, but is especially valuable in long division ,in that it permits multiplication by the figure of the quotient last obtained and subtraction of that product from the dividend or the preceding remainder to be performed in one operation. To illustrate the method, divide 387.849 by 123.456 to six figures. The mental operations are:—First figure of quotient is 3.
$3\times6=18;+1=19$ (set down 1).
$3\times5=15;+1$ (carried)$=16;$
$+9=25$ (set down 9).
$3\times4=12;+2$ (carried)$=14;+4=18$ (set down 4). Etc., etc.

Divisor	123.456
Dividend	387.849
First remainder	174910
Second remainder	51454
Quotient	3.14159

The first remainder is therefore 174910, after bringing down a zero from the dividend. The second figure of the quotient is 1. A second remainder, obtained in the same way as before, turns out to be 51454, and this divided by 12345 (using the chart) gives 4159. The complete quotient is therefore 3.14159, the absence of zeros being confirmed by a rough division with the chart, which gives 3.142. The pencil work needed is all shown above at the right It will be noticed that the economy of space compared with that needed for ordinary long division is quite as striking as the saving in time. This is a matter of some importance when a great deal of calculation needs to be done, for the reason that it permits the work to be more readily reviewed in searching for errors.

When two numbers are to be multiplied together by the precision method it is often a matter of indifference which of them is sought on the principal scale and which on the inclined scale. But since it is easier to multiply mentally by 1, 2, or 0 than by any other figures, any number beginning with these should be the one whose first figures are used separately as multipliers of the other number, and whose residue is afterward sought on the scales at the right.

Consider for example the multiplication of 7.854 by 2.143 to six significant figures. We chose here to multiply 7854 separately by 2 and 1, since that is more convenient than to multiply 2143 separately by 7 and 8. The work is completed by finding the product of 7854, sought on the principal scale, by the residue of the multiplier (4300) sought on the scales at the right, and is checked by the direct multiplication of 7854 by 2143, which gives a result correct to four figures.

But where one of the two numbers to be multiplied together is considerably shorter than the other, it is best to multiply the shorter number by the first two digits of the longer number, taken separately. For example, in the multiplication of 74.3214×26.3 to six figures, to multiply the longer number separately by 2 and 6, the first two figures of the shorter number, would mean twelve mental multiplications in all of a figure by a figure A better way is therefore to multiply 263 separately by 7 and 4, the first two figures of the longer number, which means only six

multiplications of a figure by a figure, and then add in the product of 26.3 by 0.3214, using the chart.

Squares and Square Roots

Near the left-hand margin of each section of the chart is a scale marked S, which gives the squares of quantities that appear on the principal scale. As an example of its use, let us find the square of 6.264. We connect the point 62.4 on the principal scale with the point 00 on the inclined scale, by means of the hair-line on the transparent strip. Where the scale S is intersected is read 392.4. The decimal point is best located by inspection. Since $6\times6=36$, the result must evidently be pointed off to read 39.24. In a similar way the square of 0.021 (section 7) is 0.000441, the decimal point being put where it is because $0.02\times0.02=0.0004$.

If the reader wishes a definite rule to locate the decimal point in such examples as this, he may note that the number of figures that precede the decimal point or the number of zeros that immediately follow the decimal point of the square of a number is just twice the number of figures that precede or the number of zeros that immediately follow the decimal point of the number itself; provided three figures appear in the number printed beside the scale of squares just below the place where the result is read; otherwise the decimal point of the result must be moved one place to the left. This is in fact the rule formerly given for the location of the decimal point in multiplication.

The extraction of square roots is the converse of the process just described. Point off the number whose square root is to be extracted in groups of two figures each, beginning at the decimal point Turn to a section of the chart in which the figures printed along the S scale are approximately *ten times* the left-hand group of figures (not both zeros). Pass the hair-line of the transparent strip through the point 00 on the inclined scale and the point on the S scale corresponding to the number whose square root is to be extracted. The required square root is then read from the principal scale at the left.

As an example, let us extract the square root of 3923.8. Pointing this off in groups of two figures each, beginning at the decimal point, we have 39'23.80. Ten times the first group is 390, which appears beside scale S on section 15. Connect 392.4 with the point 00. The principal scale is intersected in the point 626.4. Now each group of figures in the original number will give a figure preceding the decimal point in the answer. Since there are two groups of figures preceding the decimal point in 39'23.80, the required square root is 62.64.

To extract the square root of 0.0004411, point this off to read 0.00'04'41'10. The left-hand group of significant figures is 04. Ten times this number is 40, which appears beside the S scale on section 6. Connect the point 44.11 with the point 00. The principal scale is intersected at the point 210. Now each group of figures in the original number will be represented by a figure in the answer, which will be a zero when both figures in the group are zeros. The square root of 0.00'04'41'10 must therefore be 0.021+.

In extracting square roots, be sure to enter the chart with *ten times* the first group of figures. In the last example, if the chart has been entered with 441.1 intsead of 44.11 an incorrect answer would have been obtained.

Any round multiple of the square-root of a number may be read directly from the chart as easily as the square-root itself. Thus to find $7\times\sqrt{3923.8}$, point off this number as before, and find 392.4 on scale S. Connect this with the point 00 of the inclined scale. Where the intermediate scale numbered 7 is intersected may be read the result 438.5, the decimal point being located by inspection. Note that this

method will serve only for *round* multiples of square roots, for the reason that it is necessary for the transparent index to pass through the point 00 at the moment the result is read.

The direct use of the chart, as just described, will give square roots to four figures, with a little uncertainty in the fourth place. A five-place logarithmic table will give five figures. If a still higher degree of precision is wanted, divide the approximate square root, read directly from the chart, into the given number; then take an average of this quotient and the approximate square-root. Thus the square-root of 3923.8, read from the chart, is 62.64; and 3923.8÷62.64=62.64048, using the precision method of division. The average of 62.64 and 62.64048 is 62.64024, which is the corrected square-root, accurate to seven figures.

Problems

Square the following numbers:—

(21)	144.0	(23)	0.8765
(22)	456.0	(24)	0.04321

Extract the square root of the following numbers:—

(25)	5.00	(27)	0.030103
(26)	0.5	(28)	0.0030103

Cubes and Cube Roots

The scale marked C, on each section of the chart, is used for finding the cubes of numbers given on the principal scale. Thus to find the cube of 72.64, connect the corresponding point on the principal scale (section 16) with the point 00 on the inclined scale. The scale C is intersected in the point 383.3. To locate the decimal point, note that the number of figures that precede the decimal point or the number of zeros that immediately follow the decimal point in the cube of a number is three times the number of figures that precede the decimal point or zeros that immediately follow the decimal point in the number itself; provided that three figures appear in the number printed beside scale C just below the point where the cube is read. If less than three figures appear in that number, the decimal point will have to be moved to the left, from the position it would otherwise occupy, a number of spaces equal to the figures that are lacking. Thus the cube of 72.64 will contain six figures to the left of the decimal point (there being three figures in 380, the number printed on the chart immediately below the point where the cube is read). The final result is therefore pointed off to read 383,300.

As a further example, note that the cube of 210, read directly from the chart (section 6) is 9.26. Since there are three figures to the left of the decimal point in 210, there would be nine figures to the left of the decimal point in its cube, were it not for the fact that there is only one figure in 9, the number printed on the chart just below the point where the cube is read. Two figures of the required three are lacking, hence the decimal point of the result is moved two spaces to the left from the position it would otherwise occupy, giving 9,260,000. Similarly the cube of 0.021 is 0.00000926.

The extraction of cube roots is the converse of the operation just described. Point off the number whose cube root is to be found in groups of three figures each, beginning at the decimal point. Turn to the section of the chart in which the left-hand group of significant figures appears beside scale C (*not ten times this group, as in the case of square root*). Connect the point representing the given number

(scale C) with the point 00. Where the principal scale is intersected is read the value of the required cube root.

The decimal point is located by noting that each group of figures in the number whose cube root is to be found will give a figure of the required root, which will be a zero if a group of three zeros immediately follows the decimal point of the given number.

To find the cube root of 0.0003833, for example, point off the number to read 0.000'383'300. The first group of figures is found beside the C scale on section 16. Connecting the point 383.3 with the point 00, the principal scale is intersected at 726.4. Since there is one group all zeros immediately following the decimal point the original number, there will be one zero immediately following the decimal point of the root, which is therefore pointed off to read 0.07264. On the other hand, the cube root of 0.3833 is 0.7264.

Note that care must be taken to point off correctly the number whose cube root is to be extracted, and to enter the chart with the correct value of the left-hand group of figures. Thus, in the last problem, if the chart had been entered with the value 3.833 or 38.33 instead of 383.3, an incorrect answer would have been obtained.

To secure the cube-root of a number more accurately than this may be read from the chart, resort may be had to logarithms. Or let a be the approximate cube-root, read directly from the chart, of a number n. The corrected cube-root, accurate to seven figures, is then $\frac{1}{3}(n/a^2+2a)$.

Of course the precision methods of multiplication and division need to be used in this work, all results being carried out to seven figures. Thus the cube root of 96 is given by the chart as 4.579. The corrected root is then $\frac{1}{3}[(96 : 4.579^2)+4.579+4.579] = 4.578857$. The method, while giving results fully as accurate as a seven-place logarithmic table, is but little inferior to the latter in speed. It has the advantage of being partially self-verifying, since the approximate and corrected cube-roots must agree to three or four figures. If the method looks complicated, compare it with that taught in the grammar-schools.

Problems

Cube the following numbers:—

(29) 1.913
(30) 4.121
(31) 8.879
(32) 0.017

Extract the cube root of the following:—

(33) 0.0013
(34) 0.0130
(35) 0.13
(36) 9,630,200

Complex Operations

When the product of two numbers needs to be multiplied or divided by a third, one simply turns to a section of a chart where the product last found occurs on the principal scale, and uses this product as a new multiplicand. In this respect the nomon is vastly more convenient than any of the expensive calculating machines on the market. In the latter a product read from the machine needs to be cleared out of the product dials by turning a crank or depressing a lever, and is then set up as a multiplier by a set of keys or levers, before the second multiplication can be undertaken. For this sort of work, very common in most kinds of business, the best of the calculating machines cannot approach the speed of the comparatively inexpensive chart, which gives results more than sufficiently accurate for most kinds of work. Of course a slide rule is especially adapted for continued multiplication and division; yet because this instrument has an accuracy averaging only about one-

fifth that obtained in the direct use of the chart, the errors of operation accumulate to such an extent as to make it useless for anything but very rough work.

In carrying out a set of continued multiplications and divisions by means of the chart, it is best to take the product of all the multipliers first, then the product of all the divisors, and afterward divide the first result by the second. Thus to find the result of

$$4.27 \times 7.53 \div 33.4 \div 0.0925 \times 1.74 \div 3.1416,$$

we should arrange the work thus:—

$$\frac{4.27 \times 7.53 \times 1.74}{33.4 \times 0.0925 \times 3.1416} \quad \frac{55.94}{9.8} = 5.708.$$

In locating the decimal point of the results of operations such as this, it is best to make a rough calculation. Thus, in the preceding example, we have approximately:—

$$\frac{4 \times 7 \times 2}{30 \times 0.1 \times 3} = 6.$$

This work, performed mentally, is sufficient to identify the final result as a number in the neghborhood of 6, and thus not only fixes the position of the decimal point but helps to assure the calculator that no accidental error has been committed.

Another rule, which may be preferred by some, is to take note of the number of times the results of multiplication have been read from portions of the scales labeled with less than three figures. Move the decimal points over a corresponding number of places to the left from the positions they would otherwise occupy at the end of each of the two sets of multiplications that precede the final division.

In carrying out a series of consecutive multiplications, it is best to abandon the rule formerly given, according to which the number that begins with the larger figure is the one to be sought on the principal scale. The first product read from the chart, being a number of but four figures, is easily held in mind while the page is turned to the section of the chart in which that number occurs on the principal scale, regardless of whether the multiplier following begins with a larger or smaller figure. To proceed otherwise would make it necessary to carry two numbers in the mind at once, with an increased chance of error.

When cubes, squares, or cube-roots occur in the course of a complex calculation, it is best to perform each of these operations separately, and substitute the results in the proper places before beginning the multiplications and divisions.

In operations about as complex as those illustrated above, namely in those in which about half a dozen or more multiplications and divisions occur, the final result is apt to be in error by two or three units in the fourth place; or even more if one fails to take advantage of the precision methods of multiplication and division, when multipliers occur that begin with 1 or 2. This is a degree of accuracy more than sufficient for most purposes; but in case five significant figures are needed in the final result, with an error of but two or three units in the fifth place, it is best

to make use of logarithms, the results so obtained being checked by means of the chart.

Problems

(37) Find the value of

$$\frac{143^2 \times 214.7 \times \sqrt{.00465}}{20.66 \times 17^3 \times 148.7 \times \sqrt{21.44}}$$

(38) Find the value of

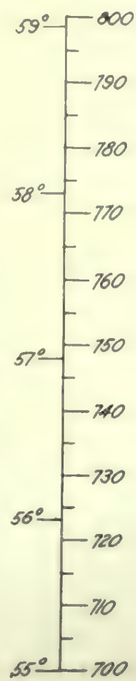
$$\frac{\sqrt[3]{.000045} \times \sqrt{.00465}}{0.1114 \times 14.14 \times .7854 \times \frac{1}{8}}$$

Special Factors

The principal advantage of the nomon over other calculating devices of comparable accuracy is to be found in the ease with which it may be adapted to special requirements. At the left of the principal scale of the ninth section of the chart will be found a mark for the special factor π. Points for the common gravimetric and volumetric chemical factors to be used as constant multiplicands, may be indicated similarly, their choice being left to the individual user, in order that the chart may not be encumbered with data of little use to him personally. Very frequently the special factors in which he is interested will all fall on a few sections of the chart, which may be purchased separately for a few cents each, thus saving wear and tear on the main set.

Special Scales

At times the special factors, laid off along the principal scale of one of the sections of the chart, are but the successive round values of a single variable, and therefore form a continuous scale. Thus we may have special scales of specific gravity, Baumé degrees, refractometric readings, temperatures, or sample weights.



The use of a special sale has a number of advantages over reference to a table. In the first place, the time needed to construct the scale is usually less than that needed to calculate a table. Second, the chart is self-interpolating. Third, the calculating device thus becomes entirely self-contained, and the chance of error that occurs in carrying data from the table to a slide rule is entirely eliminated. But perhaps most important is the fact that any change in experimental conditions generally affects all the values given in a published table and renders the latter valueless unless corrections, are applied. But if a special scale, constructed for certain conditions, is merely penciled in, it may be erased in a moment and replaced by a new one. An example will serve to indicate how such a scale may be constructed.

Let it be required to find the number of pounds of H_2SO_4 in any number of cubic feet of sulphuric acid of given Baumé reading.

Take from a table the pounds of H_2SO_4 in one cubic foot of acid, for each degree Baumé, within the range to be considered.

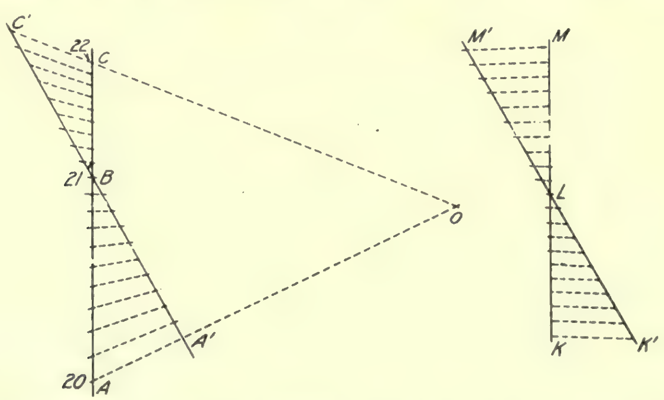
Deg. Bé.	Lbs. H_2SO_4 per cu. ft.
55	70.00
56	72.32
57	74.76
58	77.30
59	79.92

Locate a point beside the principal scale of one of the sections of the chart for each of the weights thus listed. (See annexed sketch.) If the acid ranges from 55° Bé. to 59° Bé., the special factors needed will lie beside the principal scale of section 16. But the points thus found, though located in terms of the weights of H_2SO_4 in a cubic foot of acid, are to be labeled with the Baumé readings to which they correspond. This may be expressed more formally by saying that pounds-H_2SO_4-per-cubic-foot is the *distributing function* (since the distribution of points along the special scale is thus determined) but that degrees-Baumé is the *expressed variable*. By pivoting the index over any given degree Baumé, as described under "Multiplication by a Constant", the weight of H_2SO_4 in one cubic foot may be multiplied by any number of cubic feet, to give the required total weight of H_2SO_4.

In the preceding paragraph it has been assumed that only round degrees Baumé have been located from tabulated weights per cubic foot. This is for the reason that intermediate subdivisions, representing fractions of a degree, may be found more easily by a mechanical construction, about to be explained.

Projective Scales

Let *A, B,* and *C* be the points corresponding to the values 20, 21 and 22 of the expressed variable of a special scale to be constructed. It is required to locate points corresponding to fractional values of the variable by subdividing each unit into



tenths. Through one of the three points, say the point *B*, pass an auxiliary scale, *A′C′*, making any convenient angle with *AC*, and lay off equal intervals along it, each approximately equal to one-tenth the interval *AB*. Since *B* represents 21, the tenth preceding point, *A′*, must represent 20, and the tenth following point, *C′*, must represent 22. Through *A′* and *C′* draw straight lines through the

corresponding points A and C, of the scale to be constructed. These lines will intersect in some point O. Connect O with each of the other points on the auxiliary scale $A'C'$, producing these lines, if necessary, to intersect AC. The intersections thus found are the required points of subdivision of AC. As a practical suggestion, note that if the point O tends to fall outside the limits of the drawing board, this may generally be remedied by reducing the angle between AC and $A'C'$. If the lines radiating from O intersect AC at a very sharp angle, a different length should be chosen for the subdivisions of $A'C'$.

The method that has just been described will not apply when the intervals between the successive points of subdivisioning of the scale to be constructed, and therefore the unit distances AB and BC, are very nearly equal. This is for the reason that the projection center O recedes indefinitely as the scale intervals become more nearly equal. In such cases it becomes necessary to modify the preceding construction, as shown by the figure at the right. Let KLM be the scale to be subdivided, the intervals KL and LM, which contain equal numbers of units of the expressed variable, being very nearly equal. Let an auxiliary scale $K'M'$ be constructed as before, but assume that the projection center O is in this case infinitely distant, and that the projection-lines are therefore parallel. Connect K and K', and then, with a pair of draftsman's triangles, rule lines parallel to KK' through the several points of the auxiliary scale, to intersect KL. Similarly subdivide LM by lines drawn parallel to MM'.

The expedients just described will enable one to subdivide a special scale very rapidly as soon as the points representing a few principal values have been located by calculation. The degree of approximation attained in the location of the intermediate points will depend on the nature of the data to be represented by the scale, and on the distance between the points located by direct calculation. By taking the latter near enough together, the construction may be made as accurate as one desires.

But there are many instances in which the method described is not merely an approximate construction, but is mathematically exact. This happens whenever the function whose calculated or tabulated values are used in laying off the scale (distributing function) is given by an equation of the form

$$y=\frac{ax+b}{cx+d}, \qquad (1)$$

in which x is the variable whose successive round values are given by the numerals of the scale (expressed variable), while a, b, c, and d are constants, not all unity and not all zero. It is important that the reader be able to recognize equations of this type, in order that the graphical device presented above may be used to the fullest advantage, without wasting time in calculating points that might be located mechanically. Scales constructed by this method will hereafter be referred to as *projective* scales.

As an example, let it be required to construct a special scale for calculating sucrose in raw sugar or molasses, from the change in rotation on inversion. The formula used is

$$S=\frac{P}{142.66-t/2},$$

in which P is the change in rotation due to inversion of the sample under carefully prescribed conditions, and t is the temperature of polarization. Let the reciprocal

of the quantity under the line of division be represented by T. The above equation then becomes

$$S = PT,$$

and may be solved like any other case of multiplication, as soon as a special scale of temperatures has been constructed. But the distributing function of this scale is by assumption

$$T = \frac{1}{142.66 - t/2}. \qquad (2)$$

We note that this equation is merely a special case of equation (1) above; for if $y=T$ and $x=t$, we need only give the constants a, b, c, and d in equation (1) the values 0, 1, $-\frac{1}{2}$ and 142.66 respectively, in order to have (2). The method for subdividing a special scale that has been described therefore gives mathematically accurate results in this case, and the scale of temperatures is a projective scale, which may be constructed mechanically as soon as three of its points have been located by direct calculation, no matter how far apart these three points may be. For example, one may calculate the position of the points corresponding to 15°, 25°, and 35°, those corresponding to intermediate temperatures being found by projection One need then only connect the point representing any given temperature on the special scale at the left, with the point on the inclined scale representing the change in rotation on inversion (after dropping a figure) in order to have the percentage of sucrose in the sample. The result is read from the scale that is labeled above and below with the figure dropped.

As a further example of a projective scale, consider the determination of nitrogen by the Kjeldahl method. Here the formula is

$$x = 100\, nf/w,$$

in which x is the required percentage of nitrogen, n is the difference between the cubic centimeters of alkali used in the titration of the sample and in a blank determination, f is the equivalent of each cubic centimeter of the alkali in grams of nitrogen, and w is the weight of sample. As long as the given stock of reagents lasts, f will be constant, hence the quantity $100f/w$, which we shall call W, may be treated as a simple function of the weight of sample taken, *i. e.*

$$W = 100f/w.$$

But in this equation we recognize a special form of equation (1), where $a=0$, $b=100f$, $c=1$, and $d=0$.

The scale of weights will therefore be a projective scale. If this is laid off along the principal scale, we need only connect the point representing any given weight of sample with the point on the inclined scale representing the difference between the volumes of alkali required in the titration of blank and sample (after dropping the first figure of this difference). The percentage of nitrogen is then read from one of the nine intermediate scales. It is also possible to convert any one of these scales into a double scale (like the scale of squares and cubes) in order that the percentages of nitrogen and protein in the sample may be read from the chart at the same instant. When the stock of reagents runs out, or the value of the factor f changes, one need only erase the scale of weights penciled along the left-hand margin of the chart, and put in a new scale. This takes but a few moments, since the

weight of sample used in routine work is apt to vary between quite narrow limits, or may even be constant.

Similar remarks will apply to calculations needed in the determination of the saponification numbers or iodine absorption numbers of fats or oils.

Further Remarks Concerning Special Scales

In the examples previously considered it has been assumed that any special scale to be constructed was of limited range, and might be laid off along the principal scale of one or two sections of the chart. But in many instances the distributing function has so great a range that a special scale constructed as previously described would cover too many sections to be easily executed. It may even happen that the greatest value of the distributing function is more than ten times its least value, in which case the entire set of eighteen sections furnished with this manual might appear to be of insufficient range.

This difficulty may very frequently be overcome by transformation of the given equation into some other form. If, for example, the equation presented has the form

$$P\times Q=x,$$

it also may be written

$$x\times 1/Q=P;\quad x\times 1/P=Q;\quad 1/x\times Q=1/P;$$
$$1/x\times P=1/Q;\text{ or }1/P\times 1/Q=1/x.$$

In any one of these six forms, either of the two quantities to be multiplied together may be the one sought on the principal scale, the residue of the other being found on the inclined scale. There accordingly may be as many as twelve different ways of charting any given equation, and all these possibilities ought to be separately considered whenever a simple construction is not evident at a glance.

As an illustration, consider the analysis of feeding-stuffs received in the laboratory in such moist condition that they need to be partly dried before being sampled and analyzed. Suppose the percentage of moisture (M) removed in drying varies between 40 and 60 per cent. Let it be required to recalculate the percentages (P) of the various constituents as analyzed, to percentages (x) of the original moist material. If the formula used is written

$$P=\frac{100}{100-M}\times x,$$

M will be given as a special scale, along the left-hand margin. Four sections of the chart will be needed *i. e.* sections 4 to 7, inc., the range of the distributing function, $100/(100-M)$, being from 1.667 to 2.5. Comparison with equation (1) indicates that the special scale will be projective, and may be constructed mechanically as soon as three points on each section, or twelve in all, have been located by calculation.

But another possibility is discovered as soon as we write the given equation in the form

$$(100-M)\times P=100x.$$

in this case, since the quantity $(100-M)$ ranges from 40 to 60, the scale of M will be constructed along those sections of the chart in which the range of the principal scale is from 400 to 600 (*i. e.* sections 11 to 14, inc.). But in this case the scale of M will be a uniformly divided scale, and all that we need to do to adapt the stand-

ard charts to our special problem is to substitute the values of M itself for the values of $(100-M)$ already printed along the principal scale.

Probably the quickest way to make a selection from the various methods, in cases like the above, is to write out in algebraical form the six possible variants, and then exclude from consideration those that call for the construction of special scales along the intermediate axis. Of the other forms, those that call for the construction of a special scale along the inclined axis are necessarily ruled out, unless the several values of the distributing function of that scale happen to agree in their first figures. Having thus reduced the number of variants to be considered to two or three, one needs to balance ease of construction against convenience in use, the most convenient charts to use not necessarily being those with the smallest number of sections.

The reader who is interested may secure practice in applying these principles by working out the best method for calculating the percentage of sucrose or the purity of a cane juice or syrup of given brix and polarization; or the best method for calculating the specific gravity of mineral samples by comparison of their weights in water and in air. Or let him show that it is possible to calculate the percentage of sucrose in a sample of raw sugar or molasses, from the change in rotation on inversion, by means of a special scale of temperatures, laid off along the inclined axis, instead of along the principal axis, as in the solution of this problem previously given. In this connection it is worth remarking that the condition for a special inclined scale being projective is the same as the condition noted above for projective scales along the principal axis. The inclined scale of each section of the chart is itself a projective scale.

Charts of Moderate Accuracy

There are many chemical problems of the types discussed above, for which rough computation will suffice. In these one will generally be able to make use of a nomon that covers the entire range of multiplicands from 100 to 1000 in five sections instead of eighteen. These sections are numbered to follow those accompanying the present manual.

Section Number	*Range of Principal Scale*	*Least Indicated Subdivision*
19	100-160	0.2 unit
20	160-240	0.5 unit
21	240-400	1 unit
22	400-600	1 unit
23	600-1000	2 units
24	100-600	2 units

The labor involved in the construction of special scales of extended range may be very much reduced by the use of these charts, though the accuracy of the results is correspondingly diminished. Very frequently a combination of several of these sections with those accompanying the present manual will be found to offer the best solution.

Application of the Nomon to Mixture Problems

An important application of the nomon is to be found in problems in which two ingredients are to be mixed to form a mixture of given composition; or in those in which a mixture is resolved into crystals and mother liquor, or into a concentrated solution and evaporated water Problems of this type may also be solved by means of reciprocal cross-section paper, as described hereafter; yet the latter gives some-

what less accurate results, and is intended especially for problems in which the final mixture contains more than two ingredients.

Let A and B be the weights of two ingredients to be mixed to produce a weight $A+B$ of mixture. Let a and b be the percentages in the two ingredients of some component common to both of them; and let m be the percentage of the same component in the final mixture. We shall designate a, b, and m as *composition percentages.* The best method for solving problems in which one of these five variables is to be calculated is to make use of what has been called the principle of alligation:—

The weights of two ingredients needed to prepare a given mixture are inversely proportional to the differences between the composition percentages of these ingredients and that of the mixture itself.

Formulated algebraically, this is

$$A : B :: (m-b) : (a-m), \qquad (3)$$

in which it has been assumed that m is less than a but greater than b. Another way of looking at this problem is to notice that the total weight of the given component in the mixture is the sum of the weights of that component in the two constituents. That is

$$aA+bB=m(A+B).$$

This may be transformed very readily into equation (3), which is the form that will be found most useful. If two of the five variables in the above proportion are constant throughout a series of problems, the number of variables is reduced to three, and the work may be carried out with a nomon. Several types of problems may be met, depending on which pair of variables is held constant.

1. *Final mixture of constant composition, and one ingredient of constant weight.*

Required the weight of limestone of variable $CaCO_3$ content to be added to a ton of clay, also of variable $CaCO_3$ content, to produce a cement mixture containing 75% $CaCO_3$. Here the composition percentage of the final mixture $(m=75)$ and the weight of clay $(B=2000)$ are constant. Substituting these values in equation (3) we have

$$A : 2000 :: (75-b) : (a-75),$$

or

$$[(a-75)/2000]\times A=(75-b),$$

in which a and b are the percentages of $CaCO_3$ in the limestone and clay respectively, while A is the weight of limestone needed for one ton of clay.

The last equation indicates that this problem may be solved with the nomon if values of $[(a-75)/2000]$ are located along the principal scale of one or several sections of the chart, and marked with the corresponding values of a. Thus the point corresponding to 97.2% $CaCO_3$ will be at 111, on the principal scale of section 1. To use the chart, place the index to pass through the given value of a, on the special scale thus constructed, and the value of $(75-b)$ on one of the intermediate scales. The value of A is then read at the top or bottom of this scale and on the inclined scale at the right. Thus if $a=97.2$ and $b=57.9$, we read $A=1541$ lbs. This is a result much more accurate than is needed in cement manufacture, but will serve to indicate what order of precision may be attained in work that is more exacting.

Since the composition percentage of the mixture is a round number, in this case 75.0, it is possible to replace values on the intermediate scales by values of $(75-b)$ by merely changing the numerals printed along these scales. But where the composition percentage is a mixed number, such as 75.35, it becomes impossible

to construct a special scale of b in any such simple way, and it is necessary to work out the value of $(75-b)$ for each problem separately, before entering the chart. One may also make use of a mixture diagram, as described in the next section.

When one of the two ingredients of a mixture is an acid, and the other a base, it is best to consider a, the composition percentage of the latter, as being the same as its equivalent of acid, with the negative sign prefixed. This will result in a correspondingly modified scale of a, along the principal axis.

II. Final mixture and one ingredient of constant composition.

Required the weight, A, of nitrate of soda (16% nitrogen) which must be added to a car-load of low-grade fertilizer (variable weight, B, and composition percentage, b) in order to make a mixture containing 4% of nitrogen.

Here $a=16$ and $m=4$. Substituting these values in equation (3) we have

$$A : B :: (4-b) : (16-4),$$

or

$$[(4-b)/12] \times B = A.$$

A special scale of percentages, b, may be laid off along the principal axis of an appropriate section of the chart. This adaptation is very easy in case m is a round number. Connect the given value b with the point on the inclined scale which represents B, and read the result, A, on one of the intermediate scales.

The same method will of course be used whenever a solution of variable weight and composition is diluted with water to a definite concentration.

III. Final mixture of constant composition and constant weight.

Required the weights of raw sugar and molasses, respectively, containing varying percentages of solid matter (variable brix) obtained by centrifuging one ton of mixed sugar crystals and mother liquor (massecuite) of constant brix ($m=90$). In the two preceding cases, by the construction of a special scale, we were able to avoid calculating the values of $(a-m)$ and $(m-b)$ for each separate problem. In the present case a corresponding simplification is generally impossible. The nature of the difficulty met is seen as soon as we write equation (3) in the form

$$[A/(2000-A)] \times (a-90) = (90-b)$$

Evidently this calls for the construction of three special scales in order to solve the problem directly: a special scale of A (projective) along the principal axis: a special scale of b along several of the intermediate axes, and special scale of a on the inclined axis. This construction is practicable only when m is a round number (in this case 90) in order that the scale of b may be constructed merely by altering the numerals of the intermediate scales. It is necessary also that all the values of $(a-90)$ agree in their first figure, if the problem is to be solved with a single section of the chart. If these conditions are fulfilled, the nomon furnishes a very neat and accurate solution; otherwise values of the quantities in parentheses will need to be worked out by actual subtraction in each separate problem before entering the chart to perform the final multiplication. One may also resort to a mixture diagram.

IV. Final mixture of constant weight and one ingredient of constant composition.

Let a solution of variable percentage strength, m, be evaporated to percentage strength a. Required the weight of water evaporated for each ton of original solution. Here the weight of the mixture which is to be resolved into the two in-

gredients is constant ($A+B=2000$), as is also the composition percentage of one of these ingredients ($b=0$). By substitution in equation (3), or by direct reasoning, we may obtain the formula

$$(1-B/2000)\times a=m$$

Evidently this calls for a special scale of B, but one constructed in a few moments by changing the numerals of the principal scale.

It may be noted the problem previously given, concerning the recalculation to a moist basis of analyses of samples of air dried material, illustrates the case we are discussing.

V. Final mixture and one ingredient of constant weight.

This case, which is comparatively unimportant, need not be considered in detail. Five other cases might be imagined, but these are mathematically indistinguishable from those that have been presented, and need not be separately considered.

Very frequently the two ingredients of a mixture are given in terms of their specific gravities or Baumé readings. In such cases the problem may be solved directly by the nomon, provided one of the two constituents is a solid, or a solution of constant specific gravity. We shall discuss only the most important case, which is a variant of II.

Final mixture of constant composition and one ingredient of constant specific gravity.

Let it be required to convert sulfuric acid solutions of varying volumes and Baumé readings into equivalent volumes of acid of 60° Bé. Let U be the volume of the given solution, containing S pounds of H_2SO_4 in each cubic foot. Let V be the corresponding volume of 60° acid, which contains 82.6 lbs. of H_2SO_4 in each cubic foot. Then, since these two solutions are by supposition equivalent, we have

$$SU=82.6V,$$

or

$$(S/82.6)\times U=V.$$

It is evident that this problem may be solved directly, like any other case of multiplication, as soon as a special scale has been constructed. Calculate the values of $S/82.6$, for a few round Baumé readings, within the range likely to be met in practice, and mark each with the degree to which it corresponds. By connecting the point representing any given degree with the point on the inclined scale representing U, the volume of acid given, we may read V, its equivalent in 60° acid, from one of the intermediate scales.

The reader should have little difficulty in adapting the nomon to other problems in which one of the two ingredients of the mixture is of variable, and the other of constant specific gravity. Naturally the principles that have been presented need not be limited to problems involving mixtures prepared with reference to the *weights* of some constituent. Any other property, assumed to be colligative, such as refractive index, viscosity, or optical rotation, may be used instead of composition percentage in determining the proportions in which the two ingredients need to be mixed in order to form a given mixture.

Exercises and Problems

39. The transfer of heat from circulating hot water to iron is given approximately by the equation

$$H/t=50(1.67+\sqrt{v}),$$

in which H is the number of calories of heat transferred per second per square met-

er of surface, t is the temperature difference between water and metal in degrees centigrade, and v is the velocity of the water in centimeters per second. Show how problems involving such heat transfer may be solved with a nomon. Assuming a velocity range from 40 centimeters to 2 meters per second, which sections of the chart will be needed?

40. The crystallizable sugar in sugar-cane juice is sometimes calculated by the formula

$$C=S(1.4-40/P);$$

in which C is the per cent crystallizable sugar; S is the per cent sucrose; and P is the purity of the juice. Show that the scale of purities needed to solve this problem is a projective scale. What sections of the chart should be used?

41. Show how the special scale needed in the preceding problem would be constructed.

42. In plotting the solubility curve of a salt of molecular weight M, temperatures have been laid off along a vertical axis, and grams of salt for 100 grams of solution along a uniformly graduated horizontal axis. Show that this horizontal scale may be converted into a double scale, showing not only grams of salt for 100 grams of solution, but also molecules of salt for 100 molecules of water, by the projective construction.

43. The saponification number is defined as the number of milligrams of KOH required to completely saponify one gram of a fat or oil. The standard hydrochloric acid used in the back-titration of a series of samples and blanks is of constant strength, 0.5240 $N/1$. If the titration difference between any given sample and a blank is n, the weight of the sample itself being w, derive a formula which will give s, the saponification number of the sample. If n is to be taken along the inclined scale, show that the special scale of w, which will need to be constructed along the principal axis, will be a projective scale. What sections of the chart will be needed if the weights of samples used range from 4.0 to 5.0 grams?

44. Show how one may calculate the number of gallons of water that must be added to a given weight of solution of variable percentage strength, a, in order to dilute it to a constant percentage strength, m.

45. Show how a special scale of specific gravities might be constructed, in order to read directly from the nomon the number of grams of NH_3 in any number of liters of ammonia solution of any given specific gravity.

46. Two gases of variable composition are to be mixed to form a gas of intermediate constant calorific value. Show how we may calculate the number of cubic feet of the first gas to be added to 1000 cubic feet of the second.

47. One cubic centimeter of a gas of temperature between 15° and 30° C., measured under variable pressure, is to be reduced to standard conditions. Show how this may be done directly by the nomon by means of a special scale of temperatures along the principal scale. What sections will be needed?

48. Under what circumstances may a special scale be constructed along the inclined axis of a chart? Illustrate by an example the terms *distributing function*, and *expressed variable*.

49. What is the principle of alligation? Illustrate.

50. The amount, x, by which one gram of water, weighed in air at any given temperature, exceeds one cubic centimeter, has been given by a table. Show how a special scale of temperatures may be constructed, giving the correction to be added to any given weight of water, at any temperature, in order to find its true volume in cubic centimeters. Note that by using corrections as the distributing function, instead of total values in cubic centimeters, we are able to increase enormously the accuracy of the results given by the chart. The method is analogous to that employed in precision multiplication.

51. A sugar solution, whose brix or percentage of total solids is b, is polarized without dilution, in a 20-cm. tube, showing a rotation of P degrees Ventzke. If the specific gravity corresponding to the given brix is G, derive a formula giving the percentage of sucrose, S, in the solution. Show how the nomon may be adapted to this calculation. Show how it may be made to give the purity-ratio of such a solution; *i. e.*, the ratio of sucrose to total solids.

52. The percentage of sucrose in the bagasse of cane-sugar manufacture is calculated by the formula

$$S = P \times [13(W - 50)/10{,}000];$$

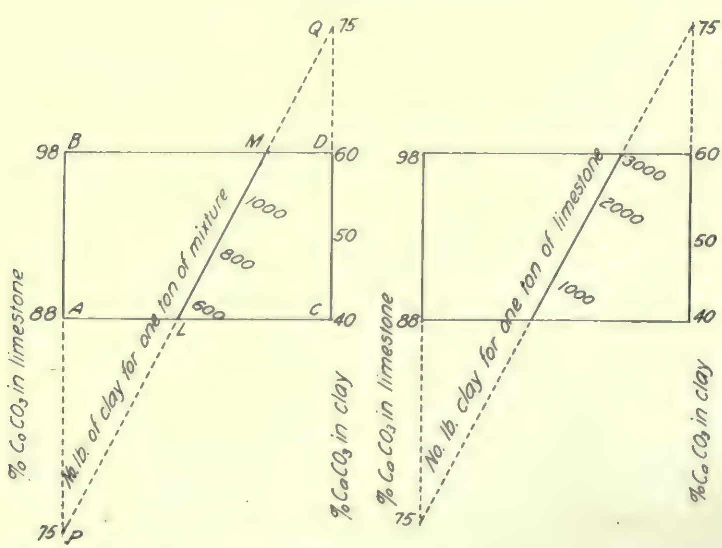
in which P is the polarization of a 100 gram sample, extracted with boiling water and polarized in a 40-cm. tube, while W is the total weight of sample plus water, after boiling. Show how this problem may be treated with a nomon. If W ranges from 400 to 500 grams, what section of the chart will be needed?

Mixture Diagrams

In the preceding pages we have shown how the nomon may be adapted to problems concerning mixtures whenever two of the variables occurring in the equation

$$A : B :: (m - b) : (a - m) \qquad (3)$$

are held constant throughout a series of determinations. In case none of the five variables is held constant, that is to say, if the problems met are entirely independent, it is possible to obtain results of moderate precision by means of a slide-rule, which is especially adapted to the solution of proportions. For rough work the extra accuracy obtainable with the nomon and the mixture diagrams about to be described would be of no importance, their chief advantage consisting in the fact that they enable one to avoid the calculation of $(m - b)$ and $(a - m)$ by separate subtractions in each individual problem, and thus tend to eliminate errors.



But in certain instances the adaptation of the nomon to mixture problems, as case III above, offers difficulties, in that the method calls for the construction of as

many as three special scales. On such occasions one resorts to mixture diagrams, whose nature may be made plain by one or two examples.

Required the proportions of clay containing a variable percentage of $CaCO_3$, which must be taken in one ton of limestone-clay mixture containing 75% $CaCO_3$.

Take two parallel axes any convenient distance apart, and graduate one in terms of the $CaCO_3$ contained in the limestone, which ranges from 88% to 98%, using any convenient unit of distance (left-hand figure). Graduate the other axis in terms of the $CaCO_3$ in the clay, which ranges, we shall say, from 40% to 60%, using the same unit distance as before, or a different one, as convenient.

The final mixture is to contain 75% $CaCO_3$. Therefore imagine the scales for $CaCO_3$ in limestone and clay to be continued downward and upward, beyond the limits of the diagram, as shown by the dotted lines in the annexed sketch, as far as the 75% point on each scale. Connect these two points with a straight line, which will intersect our rectangular drawing in the line LM. Of course in any actual case it will not be necessary to run the construction-lines over the edge of the drawing, as here shown, since one is easily able to calculate the position of the line LM. In fact, we have

$$AL=(AP\times AC)/(AP+CQ)$$

and

$$BM=(BP\times AC)/(AP+CQ).$$

Next locate, by direct calculation, three points on the intermediate scale, which is to give us the pounds of clay for one ton of mixture, assuming three round values within the range met in practice. Thus 1000 pounds clay in one ton of mixture will give us the desired composition, provided the percentages of $CaCO_3$, in limestone and clay respectively, are 90 and 60, for example. Therefore connect 60, on the clay scale, with 90, on the limestone scale, and mark the intermediate scale, at the point where it is intersected, with the corresponding weight of clay, 1000. Similarly connecting 45 (clay scale) with 95 (limestone scale) we get the point 800; and by connecting 47 with 87 we get the point 600.

The intermediate scale in all mixture diagrams is either uniformly graduated or projective. In the present case, since the distance 600-800 turns out to be the same as the distance 800-1000, it is plain that the scale is uniformly graduated. We therefore complete the construction by laying off equal intervals with a pair of dividers.

Imagine now a problem in which the weight of clay is to be determined for one ton of limestone. The construction previously used will serve, except that three points located as before will correspond to weights of clay for one ton of limestone, instead of for one ton of mixture. Let these three points be for 1000, 2000, and 3000 lbs. of clay to one ton of limestone (right-hand figure). But in this case the distance 1000-2000, on the intermediate scale, is not equal to the distance 2000-3000. We accordingly assume that the intermediate scale, instead of being uniformly divided, is a projective scale. When we have located a few points by the usual construction, one or two of them are checked by direct calculation, in order to make sure that no mistake has been made.

We shall next consider a problem in which the two ingredients of a mixture are solutions whose compositions are given in terms of their specific gravities, instead of directly in percentages. It was remarked above that the nomon can be conveniently adapted to problems of this sort only when one of the two ingredients is pure water or a solution of constant specific gravity. Required, for example, the proportions by volume in which two solutions of alcohol, of variable specific gravity, need to be mixed in order to form a solution of specific gravity 0.93.

The diagram to be constructed is very much like the two that have just been described, except that scales of specific gravities, instead of scales of percentage

compositions, are laid off along the outer axes. The distributing function of each of these scales is grams-alcohol-per-liter-of-solution. Take from a table the weight of alcohol in one liter of solution for each of a number of round specific gravities, within the ranges met in practice. Locate corresponding points along the outer axes, and mark each with the specific gravity which it represents. The intermediate scale is located by connecting the points on the other scales that represent 0.93 specific gravity. The construction is finished as previously described for cement-mixtures.

Mixture diagrams similar to the two that have just been presented might also be used in the solution of problems in which the two ingredients of a mixture are of constant composition (problems 57 and 58, below). The use of a special scale, in connection with the nomon, would, however, be equally convenient in almost every case, and decidedly more accurate.

Exercises and Problems

53. The deviation of the index of refraction of a mixture of two oils from the index of either of its constituents is known to be roughly proportional to the percentage of the other constituent in the mixture. It is required to calculate the proportions in which the two ingredients are present in a series of mixtures of intermediate specific gravity. Would a slide-rule, nomon, or mixture diagram be preferred? Why?

54. Indicate how the preceding problem would be solved by a mixture diagram.

55. Two solutions are to be mixed to form a solution of constant specific gravity. Under what circumstances would a nomon be used for routine calculations of this type, and when would it be necessary to construct a mixture diagram?

56. Fuming sulfuric acid of specific gravity 1.85 to 1.95 is mixed with acid of specific gravity 1.25 to 1.45, to produce an acid of constant strength, sp. g. 1.80. Show how a mixture diagram would be constructed to give the proportions by volume in which the two ingredients should be mixed.

57. Derive a formula giving the grams of excess SO_3 dissolved in 100 grams of fuming sulfuric acid containing various percentages of total SO_3. Show that the special scale of factors needed is a projective scale.

58. Derive a formula giving the percentage of litharge in commercial red lead containing varying percentages of lead, assuming the only oxides present to be PbO and PbO_2. Show how such a problem would be solved with a nomon.

59. Derive a formula for the weight of limestone (90% pure) that needs to be taken with one ton of quartz (98% pure) to form a silicate of the formula $n\text{CaO.SiO}_2$. Show that to solve this problem with a nomon, a projective scale of n will need to be constructed along the principal axis.

Mixtures Compounded With Respect to Two or More Chemical Constituents

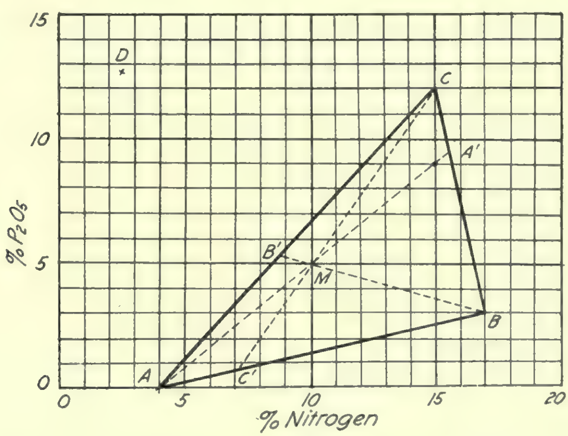
In the preceding pages we have considered mixtures whose final composition was given in terms of the percentage of a *single* constituent, such as H_2SO_4 or $CaCO_3$. But it is frequently necessary to adjust a mixture with respect to *two or more* constituents simultaneously. Thus it may be specified that a mixed fertilizer shall be compounded in such a way as to contain definite percentages of nitrogen and phosphoric acid.

Now an equation similar to (3) may be given for each of the several constituents with respect to which the final mixture is adjusted; and there may be written still another equation, which expresses the fact that the total weight of the mixture

must be the sum of the weights of its ingredients. That is to say, for a mixture compounded with respect to N chemical constituents we may write $(N+1)$ algebraical equations, sufficient to determine $(N+1)$ unknown quantities, which may be the weights of $(N+1)$ ingredients used in making up the mixture. Expressed in everyday language, *in general there must be one more ingredient entering a mixture than there are chemical constituents with respect to which the composition of the mixture is to be adjusted.*

If the composition of a mixed fertilizer, for example, is to be adjusted with respect to both nitrogen and phosphoric acid, one will generally need at least three raw materials, containing nitrogen and phosphorus. If more than three are available, there will be a number of ways of making up the mixture. However, if one of the chemical constituents of a mixture is contained in but a single ingredient, it is evident that the proportion of this constituent in the final mixture will be determined solely by the weight of that ingredient.

The algebraical methods for solving such a set of equations are too tedious to be popular, adjustment by trial being the favorite solution. But there are a number of graphical methods available, of which one will be briefly indicated. Assume that a fertilizer mixture is to be compounded to contain 10% nitrogen, and 5% P_2O_5, as indicated by the point M in the annexed figure. Let there be four raw materials, of compositions indicated by the points *A, B, C, and D.* Any three of these will in general be sufficient to compound the mixture; but it may be noted that



since the point M lies outside the area ACD, it will be impossible to compound the mixture from A, C, and D, alone. This is simply for the reason that negative weights, appearing in the algebraical solution, are not to be realized in practice.

Let the ingredients A, B, and C be selected. Draw the lines AB, BC, and CA. Connect A, B, and C with M and produce these three lines to intersect the opposite sides of the triangle. Now scale off the distances AA' and MA' with a rule divided into inches and tenths, or centimeters and tenths. The ratio MA'/AA' will be the fractional part of the final mixture that is to be made up of the ingredient A. Similarly MC'/CC' will represent the fractional part of the mixture that is C. The values of these ratios may be read from a nomon, and the results added, to give 100%, thus checking the work.

It is possible also to carry out all the above calculation graphically on the cross-section paper, but the practical advantage in so doing is not sufficient to make up for the increased complication. The most practical solution for problems in which the final mixture is adjusted with respect to more than the two chemical con-

stituents is to consider the two most important constituents first, then adjust the others by trial. The final result is to be checked by summation with reciprocal cross-section paper, as described hereafter.

Binary Scales

Each of the special scales described in the preceding pages was constructed to represent functions of a single variable, whose successive values were given by the numerals of the scale. We shall now consider binary scales; that is to say, special scales representing functions of two variables, which may vary simultaneously. The example chosen is the reduction of gas volumes to standard conditions by means of the formula—

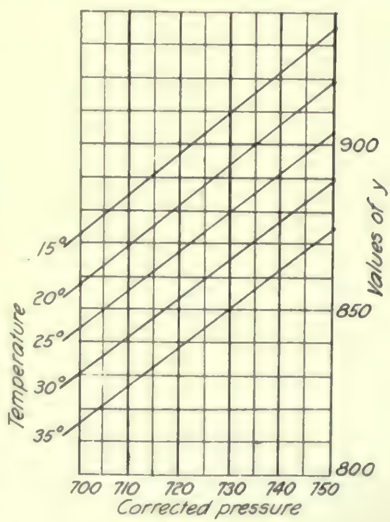
$$V_0 = V \times (273P/760T)\ ;$$

in which V is the observed volume of the gas; V_0 its volume, reduced to standard conditions; P the barometric pressure, corrected for aqueous tension; and T the absolute temperature. Let the quantity within the parenthesis in this equation be represented by y. It is evident that if we can derive its value, the problem reduces to a simple multiplication.

If the temperature range is from 290° to 305° absolute, and the range of pressures from 700 mm. to 750 mm., the value of the factor y will vary from 0.824 to 0.929. The required special scale will therefore need to be constructed along the principal scale of sections 17 and 18, or of section 23, according to the accuracy desired. The errors of ordinary analytical work, due to inaccuracies in reading volumes, temperatures, and pressures, are doubtless such that the use of a special scale in connection with section 23 would give results of sufficient precision.

The required binary scale is shown in the annexed sketch. It consists of a series of inclined lines, representing successive temperatures, across a strip of cross-sectioning in which the vertical lines represent pressures and the horizontal lines the values of the factor, y. If the tempera- is given some constant value, say 27° C. or 300° absolute, we have

$$y = 273P/228{,}000 = 0.11974P.$$



This is the equation of a straight-line. Accordingly we need locate only two points on the 27° line, one for 700 mm. and the other for 750 mm. corrected pressure, in order to have the position of that line completely determined. Proceeding in the same way with other temperatures, we construct the entire chart. To use the special scale, let a tracing of the annexed figure be pasted beside the principal scale of section 23, or let the scale be redrawn there in the proper position. Correct the observed barometric pressure by subtracting the vapor pressure indicated for the observed temperature, then enter at the top or bottom of the chart with the corrected value of P. Pass upward or downward until the proper temperature-line is met, thence horizontally to the right, where the value of the factor y is read. If the index is now pivoted over this point, the free end of the transparent strip may be swung over the face of the chart, to be multiplied by all of a series of values of V

observed at the given temperature and pressure. Volumes reduced to standard conditions are read from the intermediate scales. The reduction of gas volumes to standard conditions, and even the calculation of percentages, may also be accomplished mechanically *(Jour. Am. Chem. Soc. 39, 2145 [1917])*.

It is also possible to construct a binary scale along the inclined axis of one of the sections of the chart, if the range of the special factor is not too great. On section 24 it is also practical at times to construct special scales along the intermediate axes, thus extending the use of the nomon to equations of the form

$$P \times Q = R,$$

in which P, Q and R are each more or less complicated functions of two variables. This equation is given only to show what a chart of this type can be made to do when its possibilities are fully exploited. Equations of such a degree of complexity are rather common in engineering practice, but are seldom met in chemical work.

In the example last described, one might imagine it to be possible to construct the binary scale with vertical lines representing temperatures, and transverse lines representing pressures, thus reversing the representation of these variables adopted above. If this is done, for a constant pressure, say 760, we would have

$$y = 273/T,$$

which is the equation of a parabola, instead of a straight line, thus complicating the constuction. Before beginning a biuary scale, one should therefore take care to note whether the form in which the equation has been written is the most advantageous, and which of the two possible ways of representing the variables will lead to the easiest construction.

Probably the most important use to which binary scales are apt to be put in chemical work is in working problems in which temperature variations call for corrections in experimental data. The method applies not only to calculations carried out with a nomon, but also to tables interpolated graphically by a method described hereafter.

The Alinement-Chart Principle

We have now completed our review of the applications of the nomon to chemical calculations. In proportion as his daily work may afford the reader practice in the less simple and obvious of these applications, he will come to acquire facility in the recognition of types of problems, discrimination in the choice of methods, and skill in their execution. These are the qualities that mark the graphical expert. To those who are ambitious to progress a little further in the art of nomographic calculation, we shall now briefly indicate the general principle of the class of charts of which the nomon is but a special form, concluding with a presentation of several related methods that should prove very useful.

The graphical methods described in the preceding pages are based upon the alinement of three points, one point being selected from each of three scales. Five types of such charts are evidently possible:—

I. All three scales parallel (Triple-parallel alinement charts).

II. Two scales parallel and the third inclined (Transversal alinement charts).

III. Two scales parallel, the third being plotted along a curve (Double-parallel alinement charts).

IV. All three scales straight lines, no two being parallel.

V. Two or three of the scales curved.

We shall take up the first three of these types, which have such possibilities

of usefulness in chemical work that the reader is advised to commit to memory the three corresponding type-equations, as given below. The fourth type is simply an alternative solution of problems normally solved by the first and second types. The fifth type is of no importance in chemical work.

I. Triple-parallel charts.

An equation of the form

$$aP+bQ=R \qquad (4)$$

—in which P, Q, and R are any functions of the expressed variables p, q, and r, while a and b are constants—may be represented by a chart consisting of parallel scales of these three variables The two outer axes, those for p and q, may be any convenient distance (k) apart, the distributing functions of these scales being P and Q, respectively. Let each unit of P be represented by m centimeters and each unit of Q by n centimeters, m and n being anything convenient. The distance of the scale for r from that for p will then be:

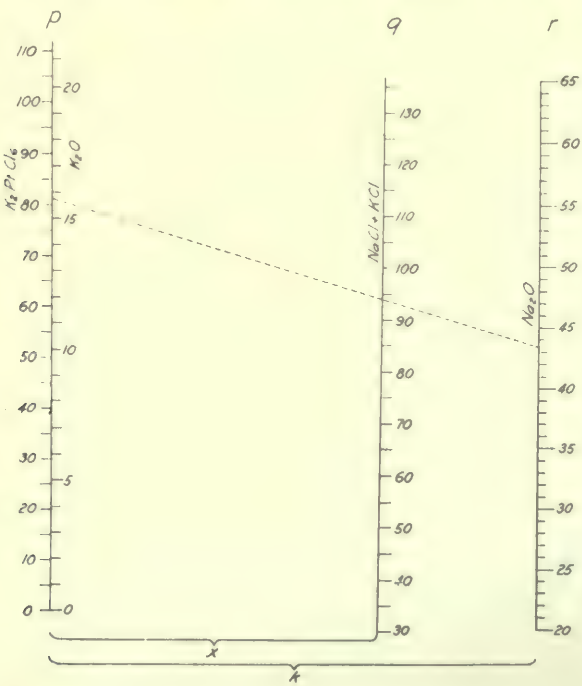
$$x=\frac{bmk}{an+bm},$$

Points along the scale for r are best located by connecting points on the two outer scales that correspond with round values of r. The following example will make this clear.

In most chemical problems the functions P, Q, and R happen to be the same as the variables p, q, and r, so that the equation given above reduces to the simpler form

$$ap+bq=r.$$

As an example, let us consider a chart for determining the milligrams of sodium oxide and potassium oxide, respectively, shown by an analysis in which the two metals



are first weighed as chlorides, the potassium being afterward collected and weighed as potassium chloroplatinate. Since the factor for converting potassium chloroplatinate into potassium chloride is 0.3068, and that for converting sodium oxide into sodium chloride is 1.583, we have

$$0.3068p+1.583q=r;$$

in which p is the number of milligrams of potassium chloroplatinate obtained, q is the number of milligrams of sodium oxide in the sample, and r is the combined weight of sodium and potassium chloride. Evidently the constants a and b in the general equation are here 0.3068 and 1.583 respectively.

Let the scales for p and r be taken 20 cm. apart ($k=20$). Unit distances, m and n, on these two scales, may be anything convenient. Thus let each unit of potassium chloroplatinate be represented by 0.2 cm distance along the left-hand axis ($m=0.2$); and each unit of sodium oxide by 0.5 cm. distance along the right-hand axis ($n=0.5$), beginning with 20 mg. (if that is the least weight likely to be met in any analysis). Then

$$x=\frac{bmk}{an+bm}=\frac{1.583\times0.2\times20}{(0.5\times0.3068)+(0.2\times1.583)}=13.47\text{ cm.}$$

Locate the middle axis at this distance to the right of the left-hand axis. Now 40 mg. of sodium chloride are equivalent to $40\div1.583=25.25$ milligrams of sodium oxide. Therefore lay a straight-edge to connect the zero of the potassium chloroplatinate scale with 25.25 on the sodium oxide scale. The point where the middle axis is crossed is marked 40. In the same way 100 mg of sodium chloride correspond to 63.16 mg. of sodium oxide. Therefore connect the zero of the potassium chloroplatinate scale with 63.16 on the sodium oxide scale, and mark the point where the middle axis is crossed 100. The distance between the points 40 and 100 on the middle scale is then divided into sixty equal parts and the graduations continued upward and downward to the limits of the chart.

As an example of the use of the chart, let us suppose that a mixture of chlorides of sodium and potassium weighs 94 milligrams, and that 81.8 mg. of potassium chloroplatinate are obtained. Connect these points on the middle and left axes, respectively, by a straight-edge; where the right-hand axis is crossed is read 43.5 mg. of sodium oxide, while just within the left margin of the chart we read 15.9 mg. of potassium oxide. The scale of potassium oxide here referred to is constructed by dividing the space between 0 and 100 of the potassium chloroplatinate scale into 19.38 equal parts, since the factor for converting potassium chloroplatinate into potassium oxide is 0.1938.

Of course the preceding chart might have been constructed with any other unit distances (m and n) for the outside axes, the middle axis being placed accordingly; and the graduations of either of the outside axes might have been begun at any convenient point. When two of the points of the middle axis have been found by calculation, intermediate points may be located by the method that has been described.

It should be noted that *all* problems based on indirect analyses, similar in principle to this, may be solved by triple-parallel charts.

II. Transversal alinement charts.

Any equation of the form

$$P/Q=R, \tag{5}$$

in which P, Q, and R are any functions of the expressed variables p, q, and r, may be solved by means of an alinement chart in which the scales for p and q are paral-

lel, and the scale for r inclined. The inclined scale may be between the two parallel scales, as in the mixture diagrams that have been described, or outside them, as in the nomon. There are in fact an infinite number of variants possible, corresponding to the infinite number of forms that the above equation may assume when written as a determinant.

If one wishes to construct a chart for an equation of the form given above, let parallel scales for p and q be taken any convenient distance apart, distances along these scales being made proportional to the functions P and Q that appear in the equation. The distances chosen to represent each unit of P and Q may be anything convenient. As for the inclined axis, this is drawn to connect the points $P=0$ and $Q=0$, which may or may not be the points corresponding to zero values of the expressed variables p and q. This inclined scale will be uniform or projective whenever R is a linear function of the expressed variable r, or the quotient of two such functions (Equation 1).

As an example, the reader may refer to the first of the charts given under the head of mixture diagrams. Here the equation is

$$(m-b)/(a-m)=A/2000.$$

Comparing this with the general equation for transversal alinement charts, we have $P=(m-b)$; $Q=(a-m)$; $R=A/2000$. Proceeding as indicated above we have precisely the construction for this chart that has been previously given.

The reader will now perceive that each section of the nomon consists of nine transversal alinement charts so superimposed that their scales for q and r coincide. It has been mentioned that an infinite number of ways for charting the equation $P/Q=R$ are indicated by nomographical theory. The principle of superposition might therefore have been applied to any one of these variants, resulting in a different type of nomon—for example one in which the intermediate scales are projective, and converge to a point beyond the limits of the section, instead of being uniformly graduated and parallel. After an experience of several years with the most promising of these variants, the writer selected the one that is here presented. By constructing the resulting chart with various degrees of accuracy, as represented by sections 1-18, 19-23, and 23-24, an instrument has been produced that can be adapted with very little trouble to the most diverse requirements.

With such a range of sections to select from, the chemist will need to construct his own transversal charts but rarely, as in certain mixture problems, previously discussed. It is believed that the directions that have been given are sufficiently detailed to enable him to work out the graphical solution of any problem that seems to require such special treatment.

III. Double-parallel alinement charts.

Any equation of the form

$$PR+QR'+R''=0, \qquad (6)$$

may be solved by an alinement chart in which the variables p and q are represented by parallel scales, while the variable r is represented by a curvilinear scale. In this equation P and Q are any functions of p and q, as before, while R, R' and R'' are all functions of the variable r. If one of these three quantities is zero, or any two of them constant, or all three of them linear functions of r, the equation can be reduced to one of the forms I or II above, and the support for the scale of r becomes a straight line. The method for constructing an alinement chart of this type may best be explained by an example.

The theoretical temperature of combustion of a fuel is given by an equation of the form—

$$at+bt^2=c;$$

in which t is the temperature in degrees centigrade, and a, b, and c are coefficients

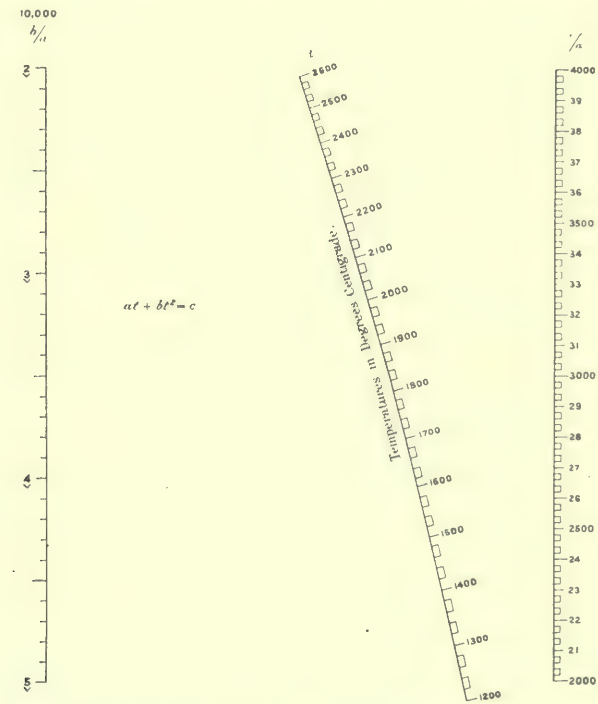
depending on the specific heats of the products of combustion, and on the quantity of air used in the burning. *(Richards, "Metallurgical Calculations," Vol. 1, p 38).*

We have here four variables, but in any given calculation we can reduce the number to three by dividing the equation through by a. It then assumes the form—

$$(b/a)t^2 - c/a + t = 0.$$

This is of the proper form for representation by a chart of the type we are considering; for $P=b/a$, $Q=c/a$, $R'=t^2$, $R''=-1$, and $R'''=t$. The supports for the scales of b/a and c/a will be parallel straight lines, while that for t will be a curve. Because R'' is negative it will be necessary to graduate one of the outer scales, say that for b/a, from above downward.

Consideration of the coefficients likely to be met in practice indicates that b/a will lie between 0.0002 and 0.005, and c/a between 2000 and 4000. Adopting pro-



per scale-units we graduate the outer supports within these respective intervals.

In locating the support for t, we must find three points, say those for 1500°, 2000°, and 2500°, by taking two pairs of values of b/a and c/a corresponding to each value of t. Thus $t=1500°$ corresponds to $b/a=0.0005$, $c/a=2625$, and to $b/a=0.0004$, $c/a=2400$. The intersection of the two straight lines connecting these values intersect in the point on the support for t that is to be numbered 1500°.

Having located three points corresponding to the three temperatures named, it will be noticed that they lie very nearly in a straight line. Because the results of a calculation of this sort are only approximate at the most, it would be proper to draw a straight line between the two extreme points that have been located and let this serve for the support for t. In the annexed figure, however, each of the principal points on the support for t has been located with accuracy by taking two pairs

of corresponding values of b/a and c/a, in order that the reader might be able to observe the exact degree of approximation to a straight line of the curve thus obtained.

Having found the support for t, we locate a few more points on it by connecting corresponding values of p and q. The subdivisions are then located with sufficient accuracy by treating the scale, section by section, as if it were projective.

Problems

60. Show how a binary scale of temperatures and specific gravities may be constructed, in order that the total weight of H_2SO_4, contained in any volume of acid, whose specific gravity is observed at any temperature between 15° and 30°, may be read directly from the nomon.

61. What are the three principal types of alinement charts? What is the general equation for each type?

62. The percentage of total solids in normal cow's milk is given approximately by the formula—

$$T=250(G-1)+1.2F+0.14$$

What type of alinement chart is adapted to the solution of this equation? The specific gravity of milk varies from about 1.026 to 1.034, and its content of fat from 3 to 5 per cent. What scale unit (cm.) must be taken in order that the scales for these variables may be respectively 24 and 20 centimeters long? If the scale of specific gravities and the scale of fats are 30 cm. apart, what will be the distance from the former to the scale of total solids?

63. The specific gravity (g) of a solid heavier than water is given by the formula

$$g=a/(a-w),$$

in which a is the weight of the solid in air, and w its weight in water. Show that this problem may be solved by a transversal diagram, the transverse scale being one of specific gravities. What special scale would be necessary to adapt this problem to calculation with a nomon?

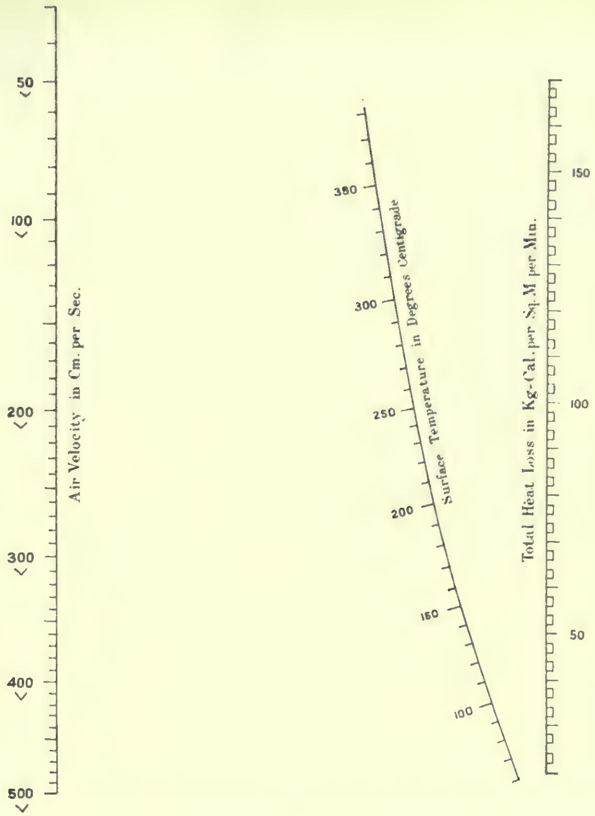
64. Show that any quadratic equation may be solved by an alinement chart of the third type (double parallel chart).

65. The rate at which heat is lost from a hot surface by convection currents of air is proportional to the difference between the surface temperature and the air temperature, and to some function of the velocity of the air current. The loss of heat by radiation is directly proportional to the difference between the fourth powers of the absolute temperatures of the radiating surface and the surrounding air, but is independent of the velocity of the air current. Show that the total heat losses from the surfaces of furnaces and other metallurgical apparatus, for constant air temperature, may be estimated by the use of alinement charts of the third type (see figure on the following page, showing the rate of loss from surfaces of oxidized iron).

66. The constant (k) of a mono-molecular reaction is given by the equation

$$k=1/t\times\log 1/(1-x),$$

in which x is the fractional part of the original material transformed in t seconds. Show how this problem may be solved with a nomon in such a way as not to re-



quire the construction of more than one special scale. Show how a transversal chart might be constructed to solve this problem.

67. Relative atmospheric humidity, R, is given by the formula

$$R = \frac{E' - 0.5(t - t')}{0.01\,E}\,,$$

in which t is the centigrade temperature of the dry bulb of a whirling psychrometer, t' is the temperature of the wet bulb, while E and E' are the respective vapor pressures of water at these temperatures. This formula applies to determinations at 755 mm. barometric pressure, uncorrected for aqueous tension; for strict accuracy the constant 0.5 should be decreased proportionately for lower pressures.

Show that it is possible to represent this equation by an alinement chart, consisting of parallel scales for R and t' and a curvilinear scale for t. When $t = t'$, we have $R = 100$. Show how advantage may be taken of this fact in locating points on the curvilinear scale.

Alinement Charts Versus Cross-Section Charts

All the problems considered in the preceding pages, for which alinement charts have been recommended, may also be solved by means of charts consisting of straight lines or curves ruled on cross-section paper. To bring out the relative

advantages of these two types of charts, it will be best to consider first an example involving but two variables.

In the analysis of coal-gas, sulfur is commonly determined as barium sulfate, which is estimated by noting the length of a column of barium sulfate suspension required to obscure a light. It is customary to show the relation between the two variables by means of a curve drawn on cross-section paper; the length of column of suspension, in millimeters, being plotted horizontally; and the corresponding weight of sulfur, in milligrams or grains, vertically. To use this chart, enter at the bottom with the observed depth of suspension, pass upward until the curve is met, thence horizontally to the right, where the weight of sulfur may be read.

Such a chart has the advantage of being quickly constructed, the printed cross-sectioning furnishing most of the diagram ready-made. It is the type to employ, whenever the data are of such a temporary nature as to make necessary frequent redeterminations of the plotted values, or whenever it is of advantage to have before one a visual representation of the way in which one of the two quantities varies with the other. But a cross-section chart has the disadvantage that it is necessary for the eye to travel a considerable distance, first vertically, then horizontally, from the place where the chart is entered to the place where the result is read. There is consequently an opportunity for error.

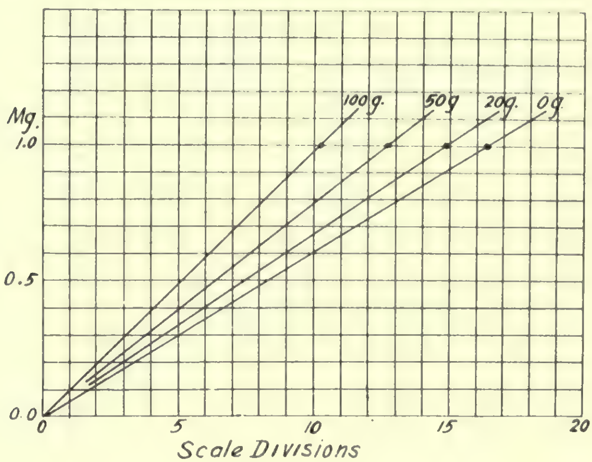
Accordingly, whenever the data to be presented by a chart for two variables are of a reasonably permanent nature, or whenever the object is accurate numerical calculation rather than graphical presentation of data, a double scale would be preferred to a cross-section chart. At the close of this manual (p. 71), for example, is a double scale of temperatures and vapor pressures of water, that permits the values of the latter to be estimated visually for each tenth of a degree from 15° to 35°. To present the same data in a table would require about two pages of print. Such a chart is constructed by laying off a uniformly divided scale of one of the variables (in this case temperature) points corresponding to round values of the other variable being located along it. Intermediate values are then found by taking the points thus located in groups of three, treating each such portion of the scale as if it were projective.

A double scale can be constructed in this way by any one who has acquired a little proficiency in the projective method, in but little more time than would be needed to plot a curve on cross-section paper. It has the advantage of being more compact than a cross-section chart, more convenient to use, and less likely to give incorrect results when hastily read. The use of cross-section charts for the representation of such data as the variation of vapor pressure with temperature, the purpose being other than rough visual presentation, is therefore to be considered as poor graphical practice, in spite of the frequency with which such charts are met.

The advantages of an alinement chart over a cross-section chart for the presentation of problems in three variables are similar to the advantages of a double scale over a cross-section chart, when but two variables are involved. The alinement chart is the more compact, and the more easily read. Then, since it may be necessary to read values intermediate between those of two neighboring curves drawn on cross-section paper, it happens that a cross-section chart gives results of lower average accuracy than does an alinement chart. *Both types of charts are possible whenever data in three variables are related as indicated by equations (4), (5), and (6) above.* We therefore find again that cross-section charts should be used to represent equations of these three types only when the purpose is visual presentation rather than numerical calculation, or when the data involved are likely to need redetermining.

Cross-section paper with special rulings, such as logarithmic paper, are now in rather common use, the object being to reduce curves to straight lines. When-

ever special cross-section paper can be purchased ready-ruled, its employment may represent an economy of effort; but *all cross-section charts consisting exclusively of straight lines can be replaced by alinement charts, no matter what the nature of the cross-sectioning employed.* We will have alinement charts of types I, II, or III, according as the straight lines of the corresponding cross-section charts are respectively parallel, concurrent in a single point, or non-concurrent. The fact that charts consisting of parallel or radiating straight lines on cross-section paper are exceedingly common, shows how general is the lack of appreciation of this principle. It requires quite special circumstances to recommend them in preference to those based on the alinement principle.



As an example of a cross-section chart that would seem to be justified by the nature of the data it represents and the manner of its employment, consider the annexed diagram, intended for use in "weighing by swings." Here we have a straight line representing the sensibility of the balance for each of a number of loads. If this diagram is posted in the balance-case, divisions of the pointer-scale may be converted directly into decimal parts of a milligram. If the sensibility of the balance changes, it is but the work of a moment to rule pencil lines, representing the new sensibility for the several loads.

Superposition of Alinement-Charts

One of the most important advantages of the alinement method is that it permits us to superimpose several charts on the same sheet of paper, in such a way as to solve a set of simultaneous equations, or a single equation involving more than three variables. An attempt to do this with a cross-section chart would result in hopeless confusion.

The annexed chart gives the number of carbon atoms (m) and of hydrogen atoms (n) for each atom of oxygen in a compound whose percentages of carbon (C) and of hydrogen (H) have been determined by combustion. It gives also the per-

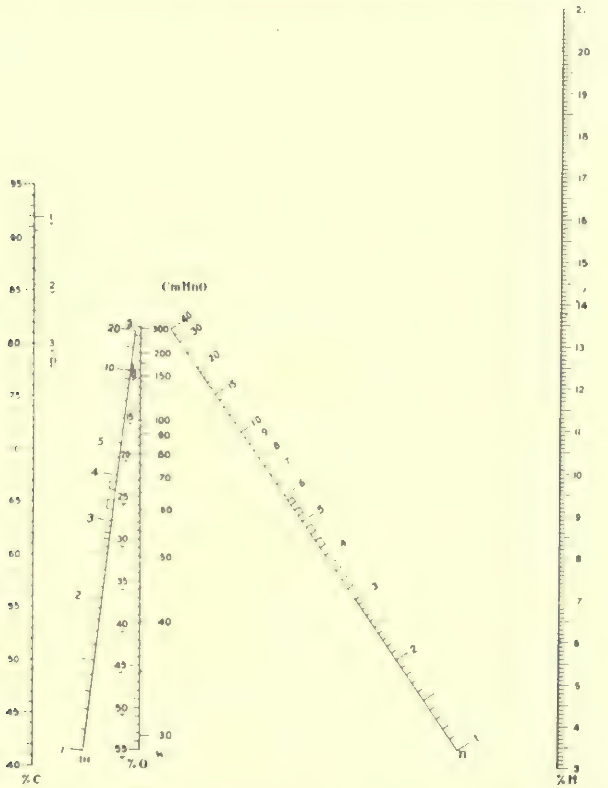
centage of oxygen (O) in the compound, and the molecular weight (W) corresponding to th formula C_mH_nO. The equations concerned are respectively

$$C/12 : O/16 = m : 1, \text{ or } 4C/3m = O;$$
$$H/1.008 : O/16 = n : 1, \text{ or } 15.88H/n = O;$$
$$C + H = (100 - O);$$

and

$$12m + 1.008n + 16 = W.$$

The first two equations may evidently be represented by transversal charts,



and the last two by triple-parallel charts, which are superimposed in such a way that the scales for the variables they have in common may coincide.

To illustrate the use of the chart, suppose that a compound contains 42.10% carbon, and 6.48% hydrogen, its molecular weight being known to be about 340. The chart gives formula $C_{1.09}H_2O$, with the molecular weight 31. Since this is about one eleventh of the known molecular weight, the formula is multiplied through by 11, giving $C_{12}H_{22}O_{11}$. The chart has been constructed for compounds containing C, H, and O alone. If other elements are present, the correct atomic ratio of C to H will still be indicated, but the rest of the formula must be derived by inspection.

Other instances in which superimposed charts might be employed occur in those analytical methods in which the components of a mixture of two reducing sugars are estimated from the reducing power and optical activity of the mixture.

Reciprocal Cross-Section Paper

In the preceding pages it has been shown that the graphical summation of two functions may be carried out with a triple-parallel chart. We may imagine a number of such charts to be superimposed, in such a way as to permit the summation of a whole series of functions. This results in what may be called reciprocal cross-section paper, in the same way that the superposition of transversal charts results in a nomon. The name reciprocal is applied to this paper for the reason that the distance from the vertical line designated by any numeral x to the right-hand margin of the paper is proportional to $1/x$.

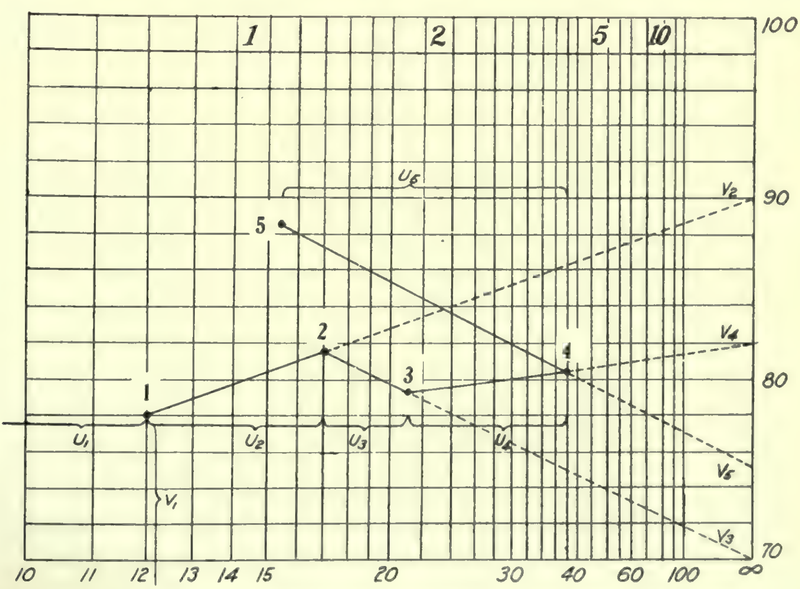
In the mixture problems that have been previously considered, it was assumed that two of the five variables occuring in such a problem were held constant, or else that a preliminary subtraction was made in order to determine the values of $(m-b)$ and $(a-m)$ before entering the nomon or mixture diagram. Such a problem may, however, be solved directly by means of reciprocal cross-section paper, which may also be used when a mixture contains several solutions of different strengths or specific gravities.

We shall consider first the use of this paper in the solution of equations of the form

$$y = \frac{u_1 v_1 + u_2 v_2 + u_3 v_3 + \ldots\ldots}{u_1 + u_2 + u_3 + \ldots\ldots\ldots},$$

which occur for example in finding weighted averages, in the calculation of the proportions in which the ingredients of a mixture need to be compounded in order to give a product of variable composition, etc.

The annexed figure shows a sheet of this paper very much reduced, with all the fine subdivisioning suppressed. In the above equation let the various u's be given



by the horizontal reciprocal scale at the bottom of the chart, and the various v's by the vertical uniform scale at the right-hand border, intersecting the horizontal scale in the point $u=\infty$. To solve the equation, begin at the point (1), whose coordin-

ates are u_1 and v_1, and pass in the direction of the point v_2 of the vertical scale, until u_2 units of the reciprocal scale have been passed over. From the point (2) thus located, pass in the direction of the point v_3 of the vertical scale until u_3 units of the reciprocal scale have been passed over. Proceeding in this way, a point is finally reached, whose ordinate is the value of y required. The figures at the top of the sheet are intended to assist the computer, by showing how many units of u correspond to each smallest indicated subdivision, in that part of the chart.

In the figure the broken line shows the solution of the equation

$$y = \frac{(12\times78)+(5\times90)+(4\times70)+(17\times82)-(22.5\times75)}{12+5+4+17-22.5}.$$

The ordinate of the last point is 88.5, the required value of y. Notice that u_5 is negative, hence the last segment of the broken line is drawn backward toward the left.

Considering *the first three terms only* of numerator and denominator, the chart gives us the averages of results that have been given the weights 12, 5, and 4 respectively; or it gives us the percentage composition of a mixture containing 1200 pounds of a 78% solution, 500 pounds of a 90% solution, and 400 pounds of a 70% solution. The chart might be used similarly in calculating mixtures of sugar-house products of varying purities, or in compounding rations. The work is most conveniently checked by repeating the construction, taking the terms in a different order.

The application of the chart just constructed to centrifugal work is very interesting. Suppose that we have 3800 lbs. of a 80.5% solution, obtained by mixing four solutions of the weights and percentages given by the broken line in the figure, as far as point (4). Suppose now that crystallization is started, and that after it is finished there is drained off 2500 lbs. of mother liquor ($u_5 = -22.5$) containing 75 % of dissolved substance ($v_5 = 75$). The chart shows that there will result 1550 lbs. of crystals, 88.5 per cent pure—point (5)—the remaining 11.5 per cent being adhering mother liquor plus water of crystallization. The same sort of chart may be used in studying the changes in the purity of sugar crystals on recrystallization or on washing in a centrifuge.

In the above diagram, the numbers along the horizontal scale are those of standard reciprocal cross-section paper as purchased. But when the range of u is different it may be necessary to multiply these values throughout by some constant. When this is done a check is furnished by the observation that the point $2u$ must bisect the interval between the point u and that marked ∞.

In the case of mixtures of the type just discussed, the graduation along the vertical scale may be taken to represent grams per liter or pounds per cubic foot, instead of percentages by weight. In this case the horizontal scale will be read in liters or cubic feet, instead of in pounds or kilos. In any case the vertical rulings may be marked in terms of specific gravities or degrees Baumé, the distributing function being grams per hundred grams, or pounds per cubic foot, according to whether the horizontal scale is to be read in weights or volumes.

The reader will also be able to extend the use of reciprocal cross-section paper to problems in which the two unknown quantities, instead of being the weight and composition of the mixture, are the compositions of two ingredients of known weight, or the weight of one ingredient and the composition of another; or in general to all mixture problems not readily adapted to calculation with a nomon for the reason that none of the variables happens to remain constant throughout a whole series of calculations.

Reciprocal paper may also be used in problems in which one needs to take the

sum of a number of terms, each consisting of the product of two quantities. Thus to solve the equation

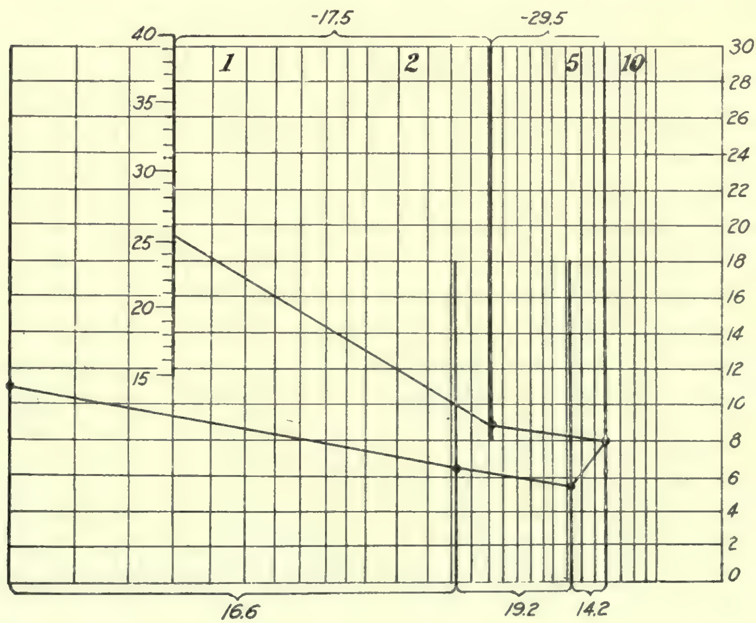
$$z = u_1v_1 + u_2v_2 + u_3v_3 + \ldots,$$

work out the value of

$$x = \frac{u_1v_1 + u_2v_2 + u_3v_3 + \ldots}{u_1 + u_2 + u_3 + \ldots}$$

by the method that has been outlined above, then multiply this result by $(u_1+u_2+u_3+\ldots)$, which may be read from the horizontal scale at the bottom of the chart just below the last point on the broken line.

This method becomes especially valuable when one of the two factors of each term is a constant. For example, in the estimating the parts per million (S) of hard scale apt to be formed by the use of boiler water of known composition, we use the formula $S = (SiO_2) + 1.66(Mg) + 1.92(Cl) + 1.42(SO_4) - 2.95(Na) - 1.75(K)$, in which (SiO_2), (Mg), etc., represent the number of parts per million silica, magnesium, etc. in the water *(Rogers, "Manual of Industrial Chemistry," 2d Ed. p. 41).* To adapt the chart to the solution of this problem, take ten times the constant multiplier in each term, *i. e.* 10, 16.6, 19.2, etc., and rule vertical lines this many units



apart, as shown in the annexed sketch. The composition of a number of samples of water may now be plotted on this one sheet. The broken line shown is for a water of the composition SiO_2, 11.0; Mg, 3.8; Cl, 4.0; SO_4, 16.0; Na, 7.0; K, 0.9. The ordinate of the final point of the broken line is 1.95. This, multiplied by 13.0, which is the value read from the horizontal scale immediately below this point, gives 25.5 parts hard scale per million. Now, in plotting the composition of any sample of water, we always come out on some point on the vertical axis through the point 13.0 on the horizontal scale. Points along this vertical axis may therefore be marked with

13.0 times the corresponding values of the axis on the right, thus forming a permanent scale from which results may be read directly. Such a chart may be used to plot a large number of analyses, before becoming illegible, if one merely marks intersections instead of drawing full lines.

Turning now to a metallurgical problem, let it be required to find the equivalent in terms of CaO of a flux containing variable percentages of CaO, MgO, FeO, Al_2O_3, and SiO_2. One pound of MgO is equivalent to 56/40=1.4 lbs. of CaO; similarly the CaO equivalent of FeO is 0.78 and that of SiO_2 is −0.93, assuming a monocalcium silicate to be formed in the slag. The negative sign indicates that the silica tends to decrease the total lime-equivalent of the flux, by uniting with the basic constituents. The sign of the lime-equivalent of the alumina is in doubt, since alumina may act either as a base or as an acid; but the general nature of the furnace charge will enable the chemist to make an assumption with regard to this point that will not be far from the truth. The equation for our problem therefore becomes

$$x=(CaO)+1.4(MgO)+0.78(FeO)+\ldots\ldots;$$

since this is of the same form as the equation for the problem last considered, it may be solved in a similar manner.

Another use of such a chart is in averaging data collected at regular intervals since the beginning of a current period. Factory records, for example, may be thus charted from the second to the twentieth, or from the fifth to the fiftieth week of the year. The vertical line for the end of the fifty-second week may be located by bisecting the interval on the horizontal scale between 26 and ∞. Such a chart will show the rate of production for each week, the average to any given date, and the rate that needs to be maintained from any date to the end of the year to make any given annual rate of production.

Other applications will doubtless occur to the reader. For example, reciprocal-cross section paper might be used for the estimation of areas by the summation of elementary polygons, or for finding the probable error when a set of experimental observations is to be treated by the method of least squares. One may also estimate stresses or moments by similar graphical methods, which possess some advantages over the so-called funicular polygon, in common use by structural engineers.

In conclusion, the attention of the reader is again invited to the fact that reciprocal cross-section paper is especially intended for use when more than two terms are to be added. When only two come into consideration, one factor of each being constant, the summation is best performed with a triple-parallel alinement chart.

Interpolation Charts

Most of the time consumed in using tables giving the relationship between two variables is in working out interpolations between the values given in the table, and most of the errors committed in the use of tables are doubtless those of interpolation. The table of logarithms that accompanies the nomon is provided with a graphical device for finding the interpolation to be added to values given by the table. This method has the advantage over the mental calculation of interpolations in that it saves time and effort, and over the use of marginal tables of interpolation in that it permits most tables to be reduced to about one-tenth of the space they would otherwise occupy.

To illustrate the method of using the chart, let it be required to find the logarithm of 3.1416. In this case the table gives log 3.14=0.49693. To find the value to be added to this in interpolation, connect 314, on the left-hand scale of the first page of the table, with 16, on the middle scale, by means of the transparent index. Where the right-hand scale is intersected will be read 22, which is the interpolation,

in units of the fifth decimal place, to be added to the logarithm read from the table, giving the corrected logarithm, 0.49715.

Since values can be interpolated accurately and instantly by this device, it follows that they need not be tabulated at such frequent intervals as would otherwise be necessary. The interval between successive values of the argument of most tables is in fact less than that needed to insure the possibility of correct linear interpolation with first-order differences, in order that the labor of interpolations may be reduced as much as possible. When the interpolation is accomplished graphically, the reason for this sort of tabulation largely disappears, and it becomes possible to reduce the length of the tables enormously. Thus the five-place logarithmic table, whose interpolation chart is here presented, has been reduced in this way from twenty pages to two, with almost no loss of accuracy. A six-place table might be reduced in a similar way from about two hundred pages to twenty, an economy truly impressive.

The preceding statements are intended to make clear the nature, advantages, and method of use of interpolation charts in general. The construction of such a chart will now be briefly indicated. Let x be the independent variable or argument of the table, and y the function of that variable whose values are given by the table for successive round values of x. Let the increment of x beyond any value given in the table be Δx, and the corresponding increment of y, which is the interpolation correction to be added to the value of y given by the table, be Δy. If the value of y is to be given by a triple-parallel alinement chart, it has been stated that an equation of the form

$$aP+bQ=R \qquad (4)$$

must exist between the three variables. In the present case, P is to be any function of x, Q any function of Δx, and R any function of Δy.

In most cases it will not be possible to show that the condition given by (4) is exactly satisfied. This will be true especially whenever the relationship between the two variables whose values are given by the table is known but empirically, as is the case when tables of specific gravities or refractive indices are concerned. In such a case one needs to be content with a practical approximation.

If the values of the independent variables are given by the table at sufficiently small intervals, the interpolation correction Δy can be assumed proportional to the increment Δx of the independent variable. In other words, $\Delta y = F . \Delta x$, in which F, the proportionality factor, depends on the tabulated value of x, that is to say is itself some undetermined function of x, which we may call X. By taking the logarithm of each side, we have then

$$\log X + \log \Delta x = \log \Delta y, \qquad (8)$$

which is evidently a special case of (4).

Having shown that the relation tabulated is of the form given by (4)—or, more commonly, having determined what is the maximum interval between consecutive values of the tabulated variable that will still permit us to assume a direct proportionality between Δx and Δy, we need only construct parallel scales of the three variables x, Δx, and Δy. The scales for Δx and Δy will be simple logarithmic scales, and may be constructed with very little trouble, round values being located with the aid of a table of logarithms. Intermediate points are then found by taking neighboring principal points in groups of three, treating this range as if it were a projective scale. Having constructed two of the three scales in this way, the scale for x, whose distributing function is the undetermined function X, may most easily

be laid off by connecting points on the other two scales that correspond to selected round values of x.

Any one of the three variables x, Δx, and Δy may be represented by the middle scale, but it is most convenient to select the variable (if any) that happens to have a negative sign when appearing in the left-hand member of (4) or when transposed to that side of the equation. Any distance may be selected to represent a unit of any of the functions determining the distribution of the points on the three scales, but if the middle scale is to be mid-way between the others, it will be necessary for the unit distances adopted for the latter to be equal. Unit distance along the middle scale will then be just half the unit distance of either of the others. If this course is adopted the three scales will not usually be of equal length. Since the ratio of unit distances on the three scales is fixed, their actual magnitudes will depend on the unit that needs to be adopted for the longest scale, in order to bring it within the limits of the page.

An obvious extension of the above method, in the case of tables of specific gravities and refractometric indices, would be to let one scale be a binary scale of the variable x and the temperature t. This would permit tabulated values of specific gravities and refractive indices to be corrected for variations from the standard temperature at the moment the interpolation is read.

It may be remarked that equation (8) above was derived from the equation $y = F.\Delta x$ by taking the logarithm of each side of the latter. In fact any equation of proper form for solution by a nomon or transversal chart could be solved by a triple-parallel chart, if first subjected to a logarithmic transformation. Instances in which this would be an advantage occur in engineering practice, but the only case in which the method seems likely to be valuable in chemical work is that in which tabulated data are to be interpolated.

Conclusion

Though the devices discussed in the preceding pages are general enough to include almost all of the calculations met in routine chemical work, there are a number of other graphical methods of considerable interest and importance. Such are those for the solution of problems connected with indirect analyses, for deduceing the order of chemical reactions, for the presentation and interpretation of experimental data involving more than three variables, and for the solution of problems in chemical thermodynamics. Even topics not strictly chemical, such as the use of reciprocal paper in the elimination of variables from a set of linear equations, or the various graphical substitutes for the method of least squares, are perhaps of sufficient importance to many persons engaged in chemical research to justify their inclusion in a work of this kind.

All these methods have however been excluded for the reason that it was impossible to tell in advance how many chemists would desire to pursue the subject of Chemical Nomography beyond its elementary stages. If any considerable measure of interest in these supplementary topics is indicated by the chemical public, it will be possible to include some of them in our next edition. In the meantime nothing is lost by the opportunity afforded to test these methods further. That such experimental work results in progress is indicated by the fact that several charts published as satisfactory graphical solutions of chemical problems, before the discovery of the methods presented here, would now be cited as first-rate examples of how one ought not to proceed.

It is forseen that the readers of this manual will fall into three classes: *conservatives,* who will be well-satisfied to perform straightforward multiplications and divisions with the nomon, but who will fail to adapt the device to any special

use whatever; *pragmatists,* who will on occasion construct simple special scales, but who will reject as impracticable, or unbusinesslike, all methods calling for the construction of binary scales or other complications requiring considerable time in drafting; *enthusiasts,* who will construct the more complex types of charts, even when the saving of time in their own work appears hardly proportionate. If now the enthusiasm of this third class of readers can be capitalized for the benefit of the rest, and their more useful results published, the art will make progress. It will doubtless even occur that the conservatives and pragmatists—when some one else has done the necessary drafting—will be glad to reverse their former opinions and make use of methods they at first condemned.

In conclusion, an appeal is addressed to all readers to speak favorably of the nomographic methods that they personally find useful; and to the graphical enthusiasts to transmit information concerning special or binary scales, interpolation charts, or other devices, that they construct and would be willing to have published in future editions of this manual, for the benefit of their more conservative colleagues.

APPENDIX

Atomic Weights and Chemical Factors

1918

Recalculated and compared with previously published tables. If a reverse factor is needed, take from the nomon the reciprocal of the given factor.

Aluminum (Al)	**27.1**
$2Al/Al_2O_3$	0.5303
$Al/AlPO_4$	0.2219
$Al_2O_3/2AlPO_4$	0.4184
Antimony (Sb)	**120.2**
$2Sb/Sb_2O_3$	0.8336
$2Sb/Sb_2S_3$	0.7142
Sb_2O_3/Sb_2S_3	0.8568
Arsenic (As)	**74.96**
$2As/As_2O_3$	0.7575
$2As/As_2S_3$	0.6092
As_2O_3/As_2S_3	0.8042
$2As/Mg_2As_2O_7$	0.4827
$As_2O_3/Mg_2As_2O_7$	0.6373
Barium (Ba)	**137.37**
Ba/BaO	0.8957
$Ba/BaSO_4$	0.5885
$BaO/BaSO_4$	0.6570
$Ba/BaCrO_4$	0.5422
$BaO/BaCrO_4$	0.6053
$Ba/BaCO_3$	0.6961
$BaO/BaCO_3$	0.7771
$Ba/BaSiF_6$	0.4912
$BaO/BaSiF_6$	0.5484
Bismuth (Bi)	**208.0**
$2Bi/Bi_2O_2$	0.8965
$Bi/BiOCl$	0.8017
$Bi_2O_3/2BiOCl$	0.8942
$2Bi/Bi_2S_3$	0.8122
Bi_2O_3/Bi_2S_3	0.9059

Boron (B)	**11.0**
$2B/B_2O_3$	0.3143
B/KBF_4	0.08723
$B_2O_3/2KBF_4$	0.27755
Bromine (Br)	**79.92**
$Br/AgBr$	0.4256
Br_2/O	9.990
$HBr/AgBr$	0.4311
Cadmium (Cd)	**112.4**
Cd/CdO	0.8754
Cd/CdS	0.7780
CdO/CdS	0.8888
$Cd/CdSO_4$	0.5392
$CdO/CdSO_4$	0.6159
Caesium (Cs)	**132.81**
Calcium (Ca)	**40.07**
Ca/CaO	0.7146
$Ca/CaSO_4$	0.2943
$CaO/CaSO_4$	0.4119
$Ca/CaCO_3$	0.4004
$CaO/CaCO_3$	0.5603
Carbon (C)	**12.005**
C/CO_2	0.27281
$C/BaCO_3$	0.06082
$CO_2/BaCO_3$	0.22295
$HCN/AgCN$	0.20176
Cerium (Ce)	**140.25**
Chlorine (Cl)	**35.46**
$Cl/AgCl$	0.2474
$HCl/AgCl$	0.2544
$Cl/NaCl$	0.6066
Cl/KCl	0.4756
Cl_2/O	4.495

Chromium (Cr)	**52.0**
$2Cr/Cr_2O_3$	0.6842
$Cr/BaCrO_4$	0.2053
$Cr_2O_3/BaCrO_4$	0.2999
$Cr/PbCrO_4$	0.1609
$Cr_2O_3/PbCrO_4$	0.2352
Cobalt (Co)	**58.97**
Co/CoO	0.7866
$Co/CoSO_4$	0.3804
Copper (Cu)	**63.57**
Cu/CuO	0.7989
$2Cu/Cu_2O$	0.8882
$Cu_2O/2CuO$	0.8995
$Cu/CuCNS$	0.5226
$CuO/CuCNS$	0.6541
Cu/Cu_2S	0.7986
Fluorine (F)	**19.0**
F/HF	0.9496
$2F/CaF_2$	0.4868
$6F/BaSiF_6$	0.4076
$2HF/CaF_2$	0.5126
$2HF/BaSiF_6$	0.4292
Glucinum (Gl) (Beryllium)	**9.1**
Gold (Au)	**197.2**
Hydrogen (H)	**1.008**
$2H/H_2O$	0.11190
Iodine (I)	**126.92**
I/AgI	0.54055
Iridium (Ir)	**193.1**
Iron (Fe)	**55.84**
Fe/Fe_2O_3	0.6994
FeO/Fe_2O_3	0.8998

Lead (Pb)	**207.2**
Pb/PbO	0.9293
$Pb/PbSO_4$	0.6831
$PbO/PbSO_4$	0.7359
$Pb/PbCrO_4$	0.6411
$PbO/PbCrO_4$	0.6906
Lithium (Li)	**6.94**
$2Li/Li_2O$	0.4645
$Li/LiCl$	0.1637
$3Li/LiPO_4$	0.1798
$2Li/Li_2CO_3$	0.1879
Magnesium (Mg)	**24.32**
Mg/MgO	0.6032
$2Mg/Mg_2P_2O_7$	0.2184
$2MgO/Mg_2P_2O_7$	0.3621
$Mg/MgSO_4$	0.2020
$MgO/MgSO_4$	0.3349
$Mg/MgCO_3$	0.2884
$MgO/MgCO_3$	0.4782
Manganese (Mn)	**54.93**
$3MnO/Mn_3O_4$	0.9301
Mn/MnS	0.6314
MnO/MnS	0.8153
$Mn/MnCO_3$	0.4779
$MnO/MnCO_3$	0.6171
$2Mn/Mn_2P_2O_7$	0.3869
$2MnO/Mn_2P_2O_7$	0.4996
$Mn/MnSO_4$	0.3638
$MnO/MnSO_4$	0.4698
Mercury (Hg)	**200.6**
Hg/HgS	0.8622
$Hg/HgCl$	0.8498
Molybdenum (Mo)	**96.0**
Mo/MoO_3	0.6667
Mo/MoS_3	0.4995

Nickel (Ni)	**58.68**
Ni/NiO	0.7858
Nitrogen (N)	**14.01**
N/NH_3	0.8227
$2N/(NH_4)_2PtCl_6$	0.06310
$2NH_3/(NH_4)_2PtCl_6$	0.07672
$2N/Pt$	0.1435
$N_2O_5/2NH_3$	3.1714
N_2O_5/N_2	3.8551
Protein/N	6.25
Osmium (Os)	**190.9**
Oxygen (O)	**16.00**
O/Cl_2	0.2256
Palladium (Pd)	**106.7**
Phosphorus (P)	**31.04**
$2P/Mg_2P_2O_7$	0.2786
$P_2O_5/Mg_2P_2O_7$	0.6385
$Ca_3(PO_4)_2/Mg_2P_2O_7$	1.3662
Platinum (Pt)	**195.2**
Potassium (K)	**39.10**
K/KCl	0.5244
$2K/K_2O$	0.8302
$K_2O/2KCl$	0.6317
$K_2O/2KClO_4$	0.3390
$2K/K_2PtCl_6$	0.1608
K_2O/K_2PtCl_6	0.1938
$2K/K_2SO_4$	0.4487
K_2O/K_2SO_4	0.54055
$2K/K_2CO_3$	0.5657
K_2O/K_2CO_3	0.6816
$3K_2O/2K_3Co(NO_2)_6$	0.3512
Rubidium (Rb)	**85.45**
Selenium (Se)	**79.2**
Silicon (Si)	**28.3**
Si/SiO_2	0.4693
SiO_2/K_2SiF_6	0.2735
$SiO_2/BaSiF_6$	0.2156

Silver (Ag)	**107.88**
$Ag/AgCl$	0.7526
$Ag/AgNO_3$	0.6350
Sodium (Na)	**23.00**
$Na/NaCl$	0.3934
$2Na/Na_2O$	0.7419
$Na_2O/2NaCl$	0.5303
$2Na/Na_2SO_4$	0.3238
Na_2O/Na_2SO_4	0.4364
$2Na/Na_2CO_3$	0.4340
Strontium (Sr)	**87.63**
Sr/SrO	0.8456
$Sr/SrSO_4$	0.4770
$SrO/SrSO_4$	0.5641
$Sr/SrCO_3$	0.5935
$SrO/SrCO_3$	0.7019
Sulfur (S)	**32.06**
$S/BaSO_4$	0.1374
$SO_2/BaSO_4$	0.2745
$SO_3/BaSO_4$	0.3430
$H_2SO_4/BaSO_4$	0.4202
$SO_3/(NH_4)_2SO_4$	0.6059
$H_2SO_4/(NH_4)_2SO_4$	0.7423
Tellurium (Te)	**127.5**
Te/TeO_2	0.7994
Thallium (Tl)	**204.0**
$Tl/TlCl$	0.8519
Thorium (Th)	**232.4**
Th/ThO_2	0.8790
Tin (Sn)	**118.7**
Sn/SnO_2	0.7877
Titanium (Ti)	**48.1**
Ti/TiO_2	0.6005
Tungsten (W)	**184.0**
W/WO_3	0.7931

Uranium (U)	**238.2**
U/UO_2	0.8816
Vanadium (V)	**51.0**
$2V/V_2O_5$	0.5604
Zinc (Zn)	**65.37**
Zn/ZnS	0..6709
Zn/ZnO	0.8034
ZnO/ZnS	0.8351
Zirconium (Zr)	**90.6**
Zr/ZrO_2	0.7390

Normal Solutions

(At. Wts. of 1918)

This table gives the number of grams of the substance named in one liter of a normal solution.

Acids and Bases.

Acetic acid	$HC_2H_3O_2$	60.042	
Ammonia	NH_3	17.034	
Ammonium hydroxide	NH_4OH	35.050	
Barium hydroxide	$Ba(OH)_2$	85.693	
Barium oxide	BaO	76.685	
Calcium hydroxide	$Ca(OH)_2$	37.043	
Calcium oxide	CaO	28.035	
Carbon dioxide	CO_2	22.002	
Citric acid	$H_3C_6H_5O_7$	64.031	
Hydrochloric acid	HCl	36.468	
Nitric acid	HNO_3	63.018	
Oxalic acid, anh.	$H_2C_2O_4$	45.008	
Oxalic acid, cryst.	$H_2C_2O_4.\ 2H_2O$	63.029	
Potassium bicarbonate	$KHCO_3$	100.113	
Potassium carbonate	K_2CO_3	69.102	(M. O.)
Potassium hydroxide	KOH	56.100	
Potassium tetroxalate	$KH_3(C_2O_4)_2.\ 2H_2O$	84.725	
Sodium bicarbonate	$NaHCO_3$	84.013	
Sodium carbonate	Na_2CO_3	53.002	(M. O.)
Sodium hydroxide	NaOH	40.000	
Sulfuric acid	H_2SO_4	49.038	

Oxidizing and Reducing Reagents.

Arsenious oxide	As_2O_3	49.480
Barium thiosulfate	$BaS_2O_3.H_2O$	267.506
Ferrous ammonium sulfate	$(NH_4)_2Fe(SO_4)_2.6H_2O$	392.260
Oxalic acid, anh.	$H_2C_2O_4$.	45.013
Oxalic acid, cryst.	$H_2C_2O_4.2H_2O$	63.026
Potassium dichromate	$K_2Cr_2O_7$	49.034
Potassium permanganate	$KMnO_4$	31.606
Potassium tetroxalate	$KH_3(C_2O_4)_2.2H_2O$	63.544
Sodium oxalate, anh.	$Na_2C_2O_4$	67.005
Sodium thiosulfate	$Na_2S_2O_3.5H_2O$	248.200
Stannous chloride, cryst.	$SnCl_2.2H_2O$	112.826

Precipitating Reagents.

Ammonium sulfocyanate	NH_4CNS	76.117
Barium chloride, cryst.	$BaCl_2.2H_2O$	122.161
Potassium cyanide	KCN.	65.115
Potassium chloride	KCl	74.560
Silver nitrate	$AgNO_3$	169.940
Sodium chloride	NaCl	58.460

Conversion Factors

These factors, based on the legal equivalents in the United States, have been compiled from various sources. Each has been checked by comparison with the Smithsonian Tables or by recalculation.

The customary units of length and mass in both the United States and Great Britain are derived from the international meter and kilogram. The legal definitions of these customary units make it appear that British units of length agree with those of the United States to within one part in about 40,000, and units of weight to within one part in 2,000,000. These discrepancies are negligible for all practical purposes, hence the following table will serve for both countries. There is, however a great difference between British and American bushels and gallons, as indicated below.

The term *cubic centimeter* is employed below as a unit of capacity, when *milliliter* is meant.

The right-hand column of figures is to be taken with the headings at the top of the page, and the left-hand column with the headings at the bottom. Thus the first line is read: "To convert inches into feet, multiply by 0.08333; to convert feet into inches, multiply by 12." Notice that each section of the table is divided into three sub-sections: *Customary to customary; metric to metric; customary to metric.*

LENGTH

	To Convert	Into	Multiply by
Length			
12.	inches	feet	0.08333
5280.	feet	statute-miles	0.0001894
0.1667	fathoms	feet	6.0
1.15157	statute-miles	nautical miles	0.86838
1000.	microns	millimeters	0.001
1000.	millimeters	meters	0.001
100.	centimeters	meters	0.01
1000.	meters	kilometers	0.001
393.7	mils	centimeters	0.002540
0.3937	inches	centimeters	2.54001
3.28083	feet	meters	0.304801
1.093611	yards	meters	0.914402
0.62137	miles	kilometers	1.60935
AREA			
1.273×10^{6}	circular mils	sq inches	7.854×10^{-7}
144.	sq inches	sq. feet	0.006944
9.	sq. feet	sq. yards	0.11111
4840.	sq. yards	acres	0.0002066
640.	acres	sq. miles	0.001562
10,000.	sq. centimeters	sq. meters	0.0001
10,000.	sq. meters	hectares	0.0001
1.973×10^{5}	circular mils	sq. centimetres	5.067×10^{-6}
0.1550	sq. inches	sq. centimetres	6.4516
10.764	sq. feet	sq. meters	0.09290
1.0764×10^{5}	sq. feet	hectares	9.290×10^{-6}
1.1960	sq. yards	sq. meters	0.8361
2.471	acres	hectares	0.40469
0.3861	sq. miles	sq. kilometers	2.5900
Multiply by	**Into**	**To Convert**	

VOLUME AND CAPACITY

	To Convert	Into	Multiply by
8.0	drams	fluid ounces	0.125
1.8047	cubic inches	fluid ounces	0.55411
231.	cubic inches	U. S. gallons	0.004329
277.27	cubic inches	English gallons	0.0036066
27.68	cubic inches	pounds of water	0.036127
2150.42	cubic inches	U. S. bushels	0.000465
2218.30	cubic inches	English bushels	0.0004508
1728.	cubic inches	cubic feet	0.00057870
1.2003	U. S. gallons	English gallons	0.8331
0.1199	U. S. gallons	pounds of water	8.34
~~0.0999~~ .1000	English gallons	pounds of water	10.00
27.	cubic feet	cubic yards	0.03704
3.068×10^{-6}	acre feet	U. S. gallons	3.259×10^{5}
8.029	cubic feet per min.	U. S. gallons per sec.	0.1247
500.4	lbs. of water per min.	U. S. gallons per sec.	0.001998

1000.	cubic centimeters	liters	0.001
100.	liters	hectoliters	0.01
1000.	liters	cubic meters	0.001

0.2705	apothecaries' drams	cubic centimeters	3.69661
0.061023	cubic inches	cubic centimeters	16.3872
0.03381	fluid ounces	cubic centimeters	29.5729
0.035314	cubic feet	liters	28.317
35.314	cubic feet	cubic meters	0.028317
1.3079	cubic yards	cubic meters	0.76458
0.264178	U. S. gallons	liters	3.78533
0.31708	English gallons	liters	4.54346
0.0283774	U. S. bushels	liters	35.238
0.0114840	U. S. bushels per acre	liters per hectare	87.078
1699.02	cu. ft. per sec.	liters per min.	0.0005885
227.126	U. S. gallons per sec.	liters per min.	0.004403
Multiply by	**Into**	**To Convert**	

WEIGHT AND FORCE

	To Convert	Into	Multiply by
1.0	grains (Troy)	grains (avoir.)	1.0
24.	grains	penny-weights (Troy)	0.04167
480	grains	Troy ounces	0.002083
437.5	grains	avoir. ounces	0.0022857
7000.	grains	avoir. pounds	0.00014286
2000.	pounds	short tons	0.0005
2240.	pounds	long tons	0.0004464
0.82285	pounds (Av.)	pounds (Troy)	1.2153
0.03108	pounds (Av.)	poundals	32.175

980.7	dynes	grams	0.001020
1000.	milligrams	grams	0.001
1000.	grams	kilograms	0.001
1000.	kilograms	metric tons (milliers)	0.001

0.034285	assay tons	grams	29.167
15.4324	grains	grams	0.06480
0.03215	troy ounces	grams	31.10348
0.03527	avoir ounces	grams	28.3495
2.20462	avoir. pounds	kilograms	0.453593
1.10231	short tons	metric tons	0.907186
0.058408	grains per U. S. gal.	milligrams per liter	17.108
0.062428	lbs. per cu. ft.	kgms. per cu. meter	16.02
0.062428	lbs. per cu. ft.	grams per liter	16.02
0.0083453	lbs. per U. S. gallon	grams per liter	119.82

PRESSURE

0.006944	lbs. per sq in.	lbs. per sq. ft.	144.
0.491174	lbs. per sq in.	inches mercury 32°F.	2.03594
13.889	lbs. per sq in.	tons per sq. ft.	0.072
14.696	lbs. per sq in.	atmospheres	0.06804
0.033418	atmospheres	inches mercury 32°F.	29.924
0.9452	atmospheres	tons per sq. ft.	1.058
0.029492	atmospheres	feet of water at 62°F.	33.908
0.43302	lbs. per sq. in.	feet of water at 62°F.	2.3094
0.88080	inches mercury 32°F.	feet of water at 62°F.	1.1353
0.03118	tons per sq. ft.	feet of water at 62°F.	32.08

1.333×10^{4}	dynes per sq. cm. (bars)	cm. of mercury 0°C.	7.502×10^{-5}
13.60	grams per sq. cm.	cm. of mercury 0°C.	.07355

.014223	lbs. per sq. in.	grams per sq. cm.	70.31
0.1934	lbs. per sq. in.	cm. of mercury 0°C.	5.171
0.5353	inches of water 62°F.	mm. of mercury 0°C.	1.8682
0.4461	feet of water 62°F.	cm. of mercury 0°C.	2.2372
0.01316	atmospheres	cm. of mercury 0°C.	76.
9.870×10^{-7}	atmospheres	dynes per sq. cm. (bars)	1.0132×10^{6}
0.000968	atmospheres	grams per sq. cm.	1033.3
Multiply by	**Into**	**To Convert**	

WORK AND HEAT

	To Convert	Into	Multiply by
777.59	foot-pounds	B. T. U.	0.001286
0.0003927	horse-power hours	B. T. U.	2547.
5.050×10^{-7}	horse-power hours	foot-pounds	1.98×10^{6}
10^{7}	ergs (dyne-cm.)	joules (watt-secs.)	10^{7}
0.4266	kilogram-meters	g-calories	2.344
0.1020	kilogram-meters	joules	9.807
4.183	joules	g-calories	0.2390
3600.	joules	watt-hours	0.0002778
7.376×10^{-8}	foot-pounds	ergs	1.356×10^{7}
3085.4	foot-pounds	Kg-Calories	0.0003241
1.980×10^{-8}	foot-pounds	horse-power hours	5.050×10^{7}
0.001558	horse-power hours	Kg-Calories	641.7
3.96832	B. T. U.	Kg-Calories	0.2520
0.009302	B. T. U.	kilograms-meters	107.5
0.0009486	B. T. U.	joules	1054.
3.415	B. T. U.	watt-hrs.	0..2928
1.8	B T. U. per lb.	Kg-Calories per kg.	0.55556
0.11237	B. T. U. per cu. ft.	Kg-Cal. per cu. meter	8.899
1.341	horse-power hours	kilowatt-hours	0.7457
POWER			
550.	ft.-lbs. per sec.	horse-power	0.001818
33000.	ft.-lbs. per min.	horse-power	3.030×10^{-5}
33,520.	B. T. U. per hour	horse-power	0.0003927
558.67	B. T. U. per min.	horse-power	0.02356
1	joules per sec.	watts	1.
10^{7}	ergs per sec.	watts	10^{-7}
0.1020	kg.-meters per sec.	watts	9.807
75.	kg.-meters per sec.	cheval-vapeur	0.01333
4.1832	watts	g-calories per sec.	0.239
69.72	watts	Kg-Calories per min.	0.01434
0.12055	ft.-lbs. per sec.	kg.-meters per min.	8.2953
0.7376	ft.-lbs. per sec.	watts	1.356
1.341	horse-power	kilowatts	0.7457
0.986319	horse-power	cheval-vapeur	1.01387
VELOCITY			
1.4667	feet per sec.	miles per hr.	0.6818
1.152	feet per sec.	knots per hr.	0.5921
27.78	centimeters per sec.	kilometers per hr.	0.036
0.9113	feet per sec.	kilometers per hr.	1.097
1.9682	feet per min.	centimeters per sec.	0.5080
54.6806	feet per min.	kilometers per hr.	0.01829
0.02237	miles per hr.	centimeters per sec.	44.7
Multiply by	**Into**	**To Convert**	

TEMPERATURE

	To Convert	Into	Multiply by
1.8	deg. F. +460	deg. A.	0.5556
1.8	deg. F. −32	deg. C.	0.5556

ELECTROCHEMICAL

96,500.	coulombs	gram-equivalents	1.0411×10^{-5}
26.806	ampere-hours	gram-equivalents	0.037445
12,159.	ampere-hours	pound equivalents	8.2243×10^{-5}
0.24620	ampere-hours	grams of Ag.′	4.062
0.84336	ampere-hours	gms of Cu.″	1.1857

MATHEMATICAL

0.4343	logs to base 10	logs to base *e*	2.303
0.017453	radians	degrees	57.2958
2.9089×10^{-4}	radians	minutes	3437.7
0.016667	degrees	minutes	60.
2.7778×10^{-4}	degrees	seconds	3600.
Multiply by	**Into**	**To Convert**	

Calibration of Volumetric Apparatus

(Abstracted from Circulars 9 and 19, U. S. Bureau of Standards)

Glass apparatus to be calibrated must be so thoroughly cleaned that no drops remain adhering to the surface as the water drains away.

Flasks and cylinders used to deliver a definite volume should be emptied by gradually inclining them. When the continuous stream of water ceases, they are held nearly vertical for half a minute. The mouth is then brought into contact with the wet surface of the receiving vessel, to remove the remaining drop.

In filling a pipet or buret, any air bubbles within the outlet should be removed. The tip of the instrument should be stroked off with filter-paper after adjusting to the zero-mark.

Stopcocks of burets should be completely open during emptying. When the continuous outflow ceases, the tip should be touched against the wet surface of the receiving vessel. Transfer pipets should be held in a vertical position until the surface of the descending liquid reaches the restricted lower portion. The tip should then be touched against the wet surface of the receiving vessel, held there until the outflow ceases, then removed. The drop of water that always remains in the tip should not be blown out.

Error due to improper drainage of a buret or pipet is to be circumvented by using an instrument having a tip that tapers gradually for 2 or 3 cm., without sudden contraction at the orifice, the time of outflow being not less than 2 seconds for each cm. of linear distance along the graduated portion of the instrument, nor more than 2 mins. altogether. Transfer pipets should have tips that permit delivery in not more than one minute nor less than the following number of seconds for the indicated sizes:—

cm^3	5	10	50	100	200
secs.	15	20	30	40	50

Burets, measuring pipets, and *transfer pipets* are calibrated by discharging water into a weighing bottle provided with a rubber stopper carrying a thermometer. The temperature of the water is noted immediately after being discharged into the flask. If the instrument is marked to hold v cubic centimeters, its true volume, at a standard temperature if 20°C is then $W+va/1000$ in which W is the weight of water discharged, while a is a constant for each calibration temperature. It is convenient to use a counterpoise in this weighing.

t	15	16	17	18	19	20	21	22	23
a	2.07	2.20	2.34	2.49	2.65	2.72	3.00	3.19	3.40
t	24	25	26	27	28	29	30	31	32
a	3.61	3.83	4.06	4.31	4.56	4.82	5.08	5.34	5.61

These values assume normal atmospheric pressure, 50% relative humidity, and a cubical coefficient of expansion for glass of 0.000025. A special scale of these values may be constructed along section 21 or 22 of the nomon, points representing tenths of a degree being estimated visually or located by projection where this degree of precision is necessary.

The weight of water to be taken in calibrating a *flask* is $v-va/1000$, in which v is the indicated volume. This weight of water is to be added to a dry flask, if the latter is to be calibrated to hold v cubic centimeters; or to a wet flask which has been drained as prescribed above, if the latter is to deliver that volume. Each piece of calibrated apparatus should be given a distinctive number, and flasks

should be marked as calibrated "to contain" or "to deliver" the indicated volume.

In routine work, the capacity of flasks is best determined by filling them from a vessel calibrated to contain a certain volume slightly less than that indicated. The deficiency is then added from an accurately graduated buret.

Instruments calibrated to deliver a definite volume of water will not deliver the same volume of other liquids or of solutions. Their behavior with the latter may be determined by discharging them into another vessel of the same capacity, marking the level to which the second vessel is filled. The latter is then calibrated to that point with water.

Reduction of the Volumes of Liquids to a Standard Temperature.

If all the pieces of volumetric apparatus in use in a laboratory are calibrated as described above, their true capacity at 20° C. will be known, and they will preserve their proper relative capacities at all temperatures. Error can arise only when volumetric solutions are standardized at a temperature different from that at which they are used. For example, if a solution is made up and standardized at 28°, assuming the volumes of all the apparatus used to remain as determined for 20°, no error can result, provided all determinations in which this solution is employed are carried out at 28°. But if such a solution were to be used at 18°, an error would be introduced, which would be negligible in many kinds of work, but whose magnitude the careful analyst would take pains to estimate.

To correct any volume of water, v, from an observed temperature, t, to a temperature, t', add the quantity $v(a' - a)/1000$; in which a' and a are the corrections tabulated above for the temperatures t' and t respectively. For example a volume of 35.7 cm^3, at a temperature of 18° C. corresponds to $35.7 + 35.7(4.56 - 2.49)/1000$ at 28° C.; or, as a sufficient approximation, to $35.7 + 40 \times 2.07 \div 1000 = 35.78$ cm^3.

The above correction is for pure water. That for N/10 solutions is about 10%, for normal HCl about 20%, and for normal H_2SO_4 and the alkalies about 50% greater than the corresponding correction for water. For more concentrated solutions see the *Chemiker-Kalendar (1911)*, p. 304.

The chemical control of many industrial enterprises involves the reduction of solutions, measured hot, to a standard temperature. If the correction-factors needed for a unit volume are plotted as a special scale of temperatures along the principal axis of an appropriate section of the nomon, multiplication by any given volume of solution may be performed directly. Reduction of specific gravity readings from one basis to another may be accomplished in a similar manner.

Correction of Weights to a Vacuum

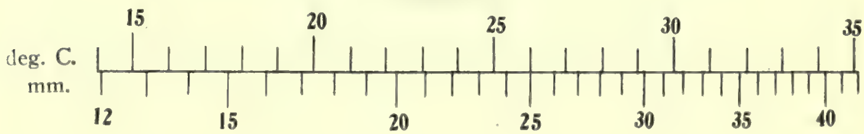
If a body of specific gravity s has an apparent mass of g grams when weighed in the air with brass weights, its weight in vacuo will be g grams plus kg milligrams, in which $k = 1.2/s - 0.15$. This formula assumes a specific gravity of 8.0 for the brass weights. The value of k will be negative if the specific gravity of the substance weighed is less than 8. The value of $1.2/s$ is read from the nomon, decreased by 0.15, and a corresponding point marked on one of the sections of the chart, to serve as a constant multiplier for a series of observed values of g.

Standardization of Thermometers

For precautions and details in determining the positions of the fixed points (0°C and 100°C) see Circular No. 8, U. S. Bureau of Standards. The correction for emergent stem in centigrade thermometers is $0.00016n(t-t')$, in which n is the number of degrees of emergent thread, t is the temperature of the bath, and t' is the average temperature of the emergent thread

Vapor Pressure of Water

(*Regnault*)



Answers to Problems

(1) 6308

(2) 222.8

(3) 0.004815

(4) 23.41

(5) 0.007789

(6) 9.488

(7) 0.002591

(8) 9,037,000

(9) 555.5

(10) 0.000003896

(11) 0.2173

(12) 989,800

(13) 3.821

(14) 0.01554

(15) 0.9896

(16) 0.3183

(17) 0.12858

(18) 6667

(19) 1.2732

(20) 0.005678

(21) 20,740

(22) 207,900

(23) 0.7682

(24) 0.001867

(25) 2.236

(26) 0.7071

(27) 0.1735

(28) 0.05487

(29) 7.000

(30) 70.04

(31) 700.0

(32) 0.000004913

(33) 0.1091

(34) 0.2351

(35) 0.5066

(36) 212.7

(37) 0.005468

(38) 0.1548

(39) 22, 23

(40) 23

(43) 14, 15, 16

(47) 1 and 2; or 17

(52) 22

(62) $m = 3000$

$n = 10$

$x = 17.7$ cm.

Price List

Nomon, General Edition..........$1.00

Sections 1 to 18, inclusive, with separately bound directions. Multiplication, division, powers, roots, logarithms. Average error about one part in four or five thousand.

Nomon, as above, without manual of directions..........$0.80

Nomon, Students' Edition..........$0.50

Sections 19 to 24, inclusive, with directions. Multiplication, division. Average error about one part in one thousand. Includes five-place table of logarithms, with directions.

"*A Manual of Chemical Nomography.*" By Horace G. Deming..........$1.25

Nomon, general edition, with supplementary graphical devices and chemical tables.

"*Nomographic Business Arithmetic.*" By Howard A. Hardy..........$1.00

Nomon, general edition, with applications to business arithmetic, interest and discount tables, etc.

Nomon, Sugar Technologists' Edition. In preparation..........$1.50

Six special charts, on bristol board, for calculating purity coefficients from brix and polarization.

The charts issued in connection with the first four of the manuals listed above are on tough ledger-paper, with specially prepared surface. They are almost as permanent as those printed on heavier stock, and have the advantage of being flexible enough to permit their being rolled up and carried in the pocket. They are printed on one side only, with marginal index, and bound at one edge. Sold separately, each..........$0.05

Sections 1 to 24, inclusive, are sold separately on reinforced bristol-board, surfaced with transparent lacquer. Printed on front and back, the sections being paired off as follows:—1,2; 3,4; 5,6; 7,8; 9,10; 11,12; 13,14; 15,16; 17,18; 19,20; 21,22; 23,24; logarithms; specific gravity table. Each sheet..........$0.10

Reciprocal cross-section paper. Sold only in blocks of 24 sheets, each block..........$0.25

Celluloid strips, ruled or unruled..........$0.10

We do our best to supply ruled strips with straight hair-lines, but cannot guarantee them.

Simplicity is Economy. Economy is Patriotism. The simplest way is to inclose cash with your order. Please add 10c extra to cover mailing costs on orders less than 50c.

THE NOMON SALES AGENCY

URBANA, ILL.

L